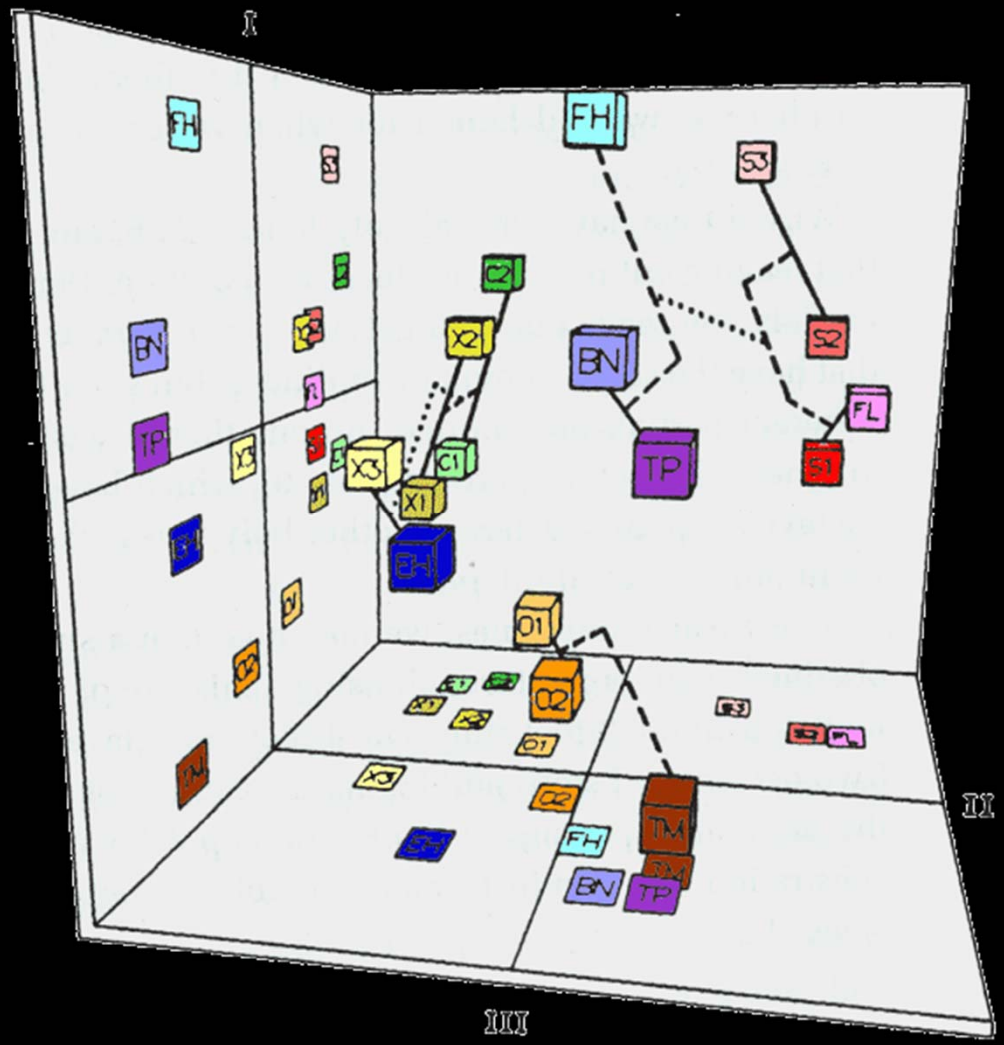
Diagrammatik der Tonhöhen

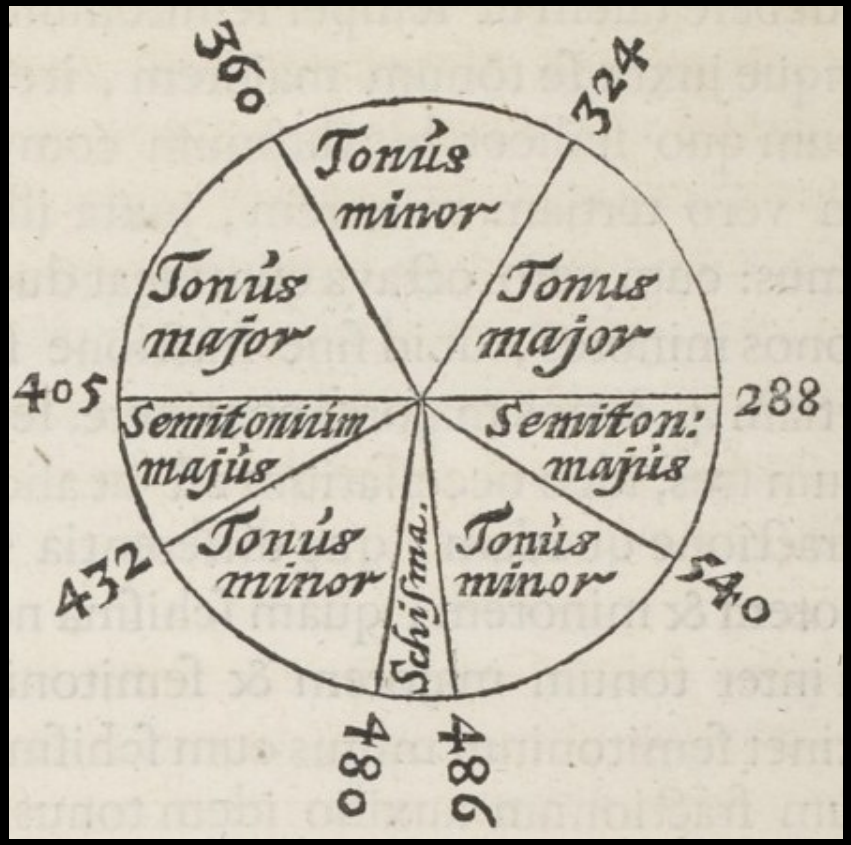
Daniel Muzzulini

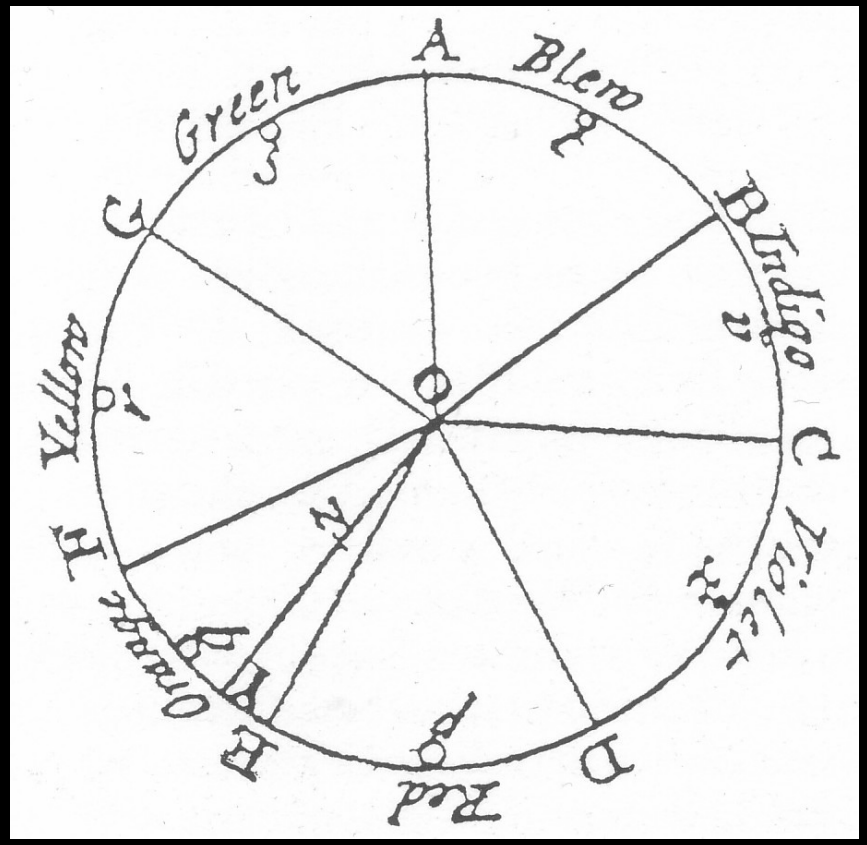
28. Oktober 2016

Klangfarbenraum – timbre space

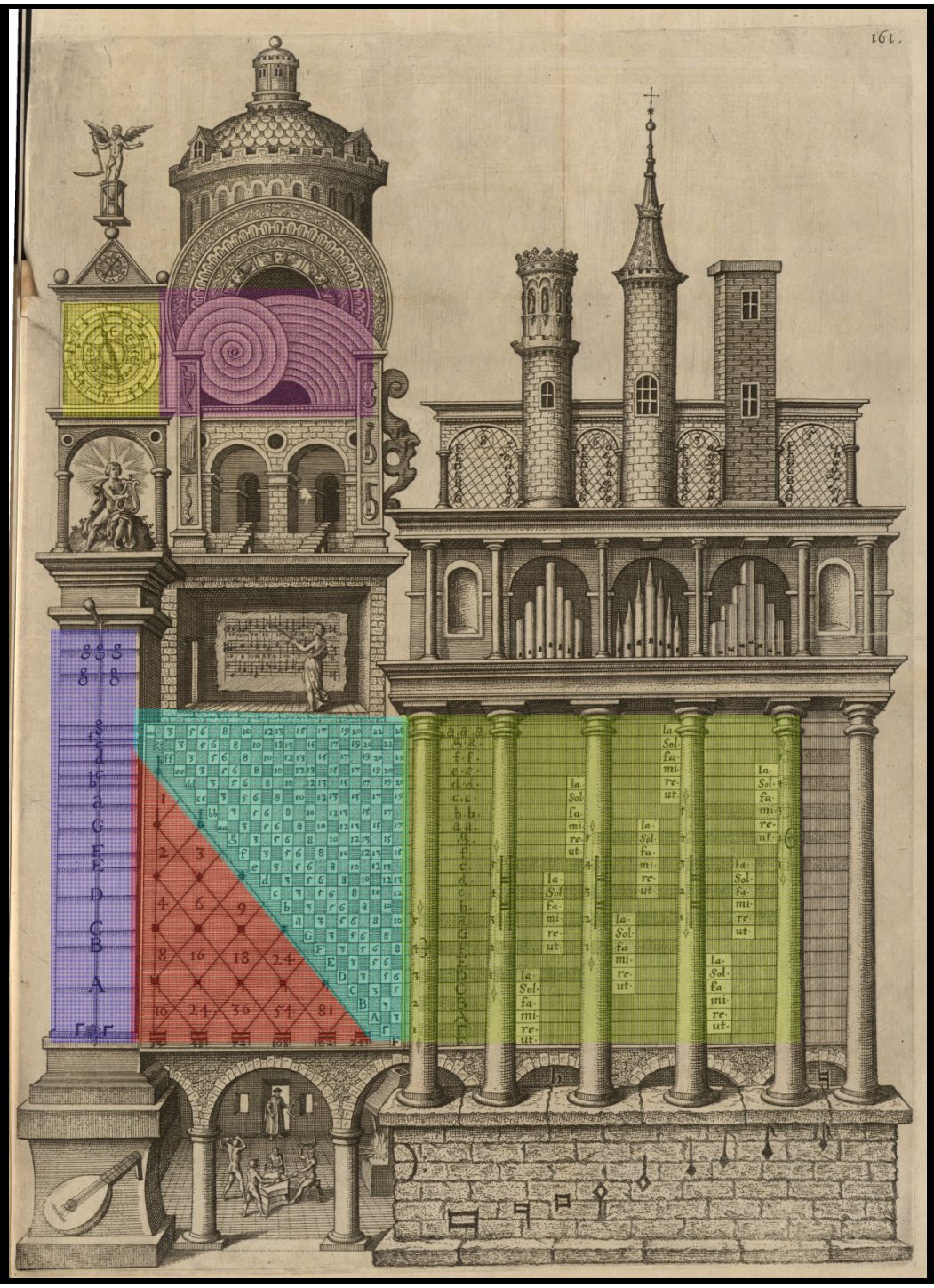


Descartes and Newton



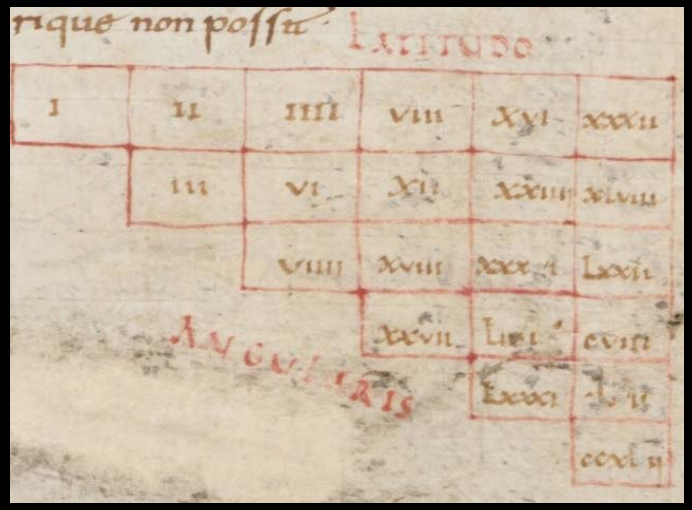


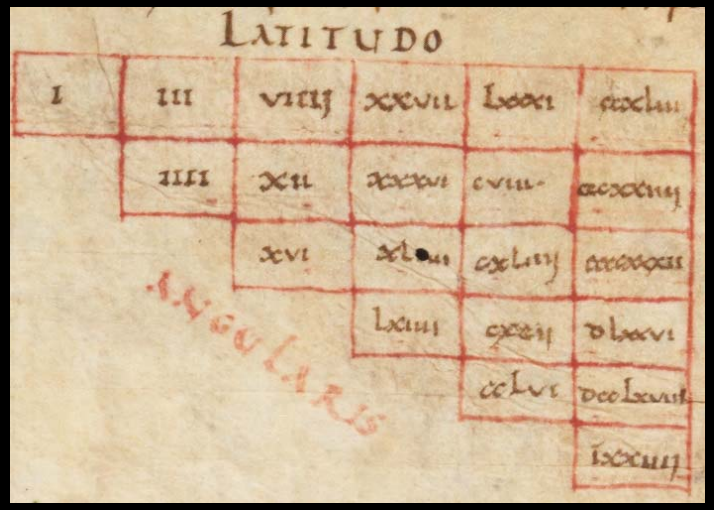
μουσειο =
Temple of the Muses



Robert Fludd (1618/24)
Templum Musicæ

Boethius: number grids

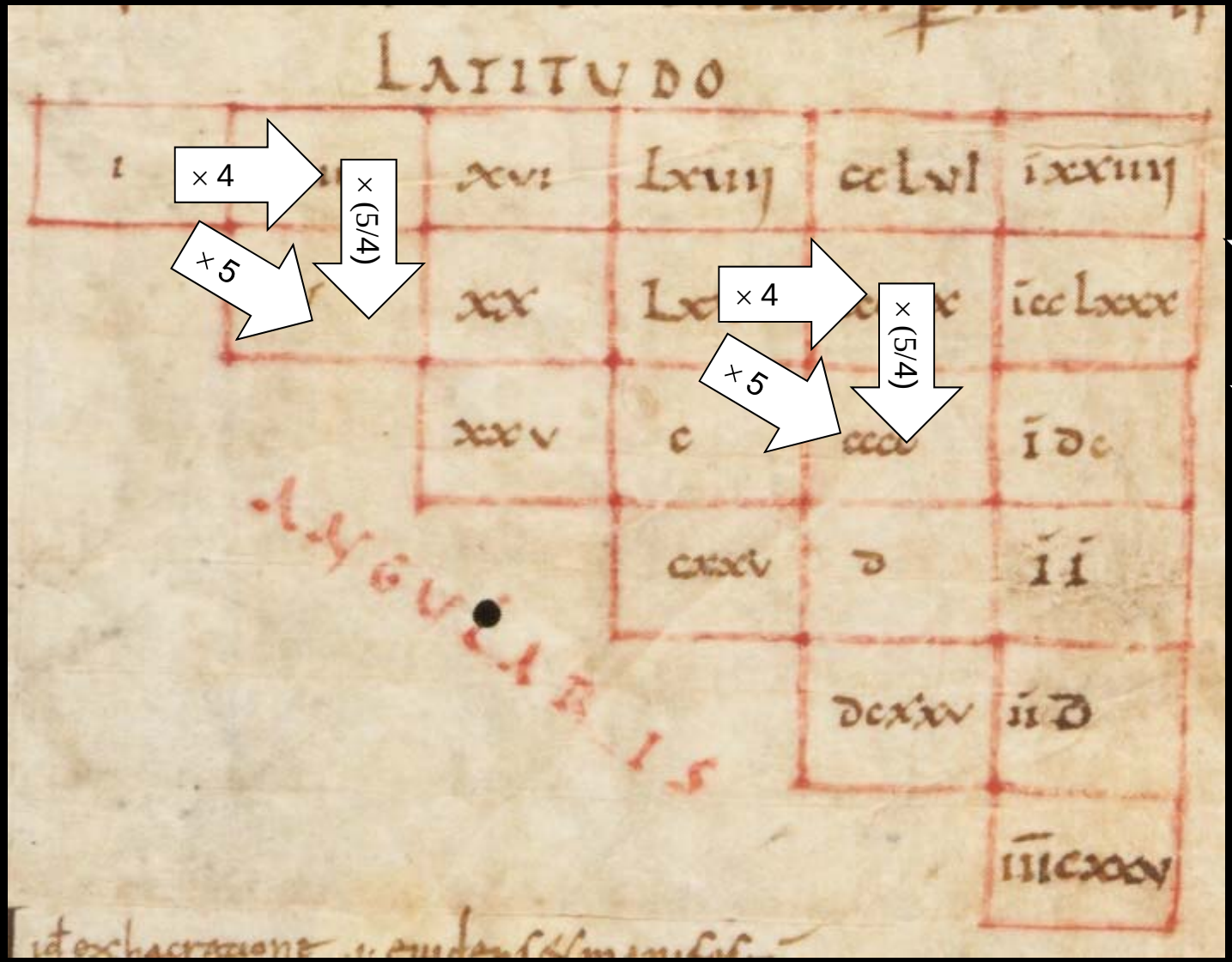




CSI:187,4

I	VIII	LXIIII	DXII	IIII. XCVI	XXXII. DCCLXVIII	CCLXII. CXLIIII
	IX	LXXVI	DLXXVI	IIII. DCVIII	XXXVI. DCCCLIIII	CCXCIIII. DCCCCXII
		LXXXI	DCXLVIII	V. CLXXXIIII	XII. CCCCLXXII	CCCXXXI. DCCXXVI
			DCCXXIX	V. DCCCXXXII	XCVI. DCLVI	CCCLXXIII. CCXLVIII
				VI. DLXI	LII. CCCCLXXXVIII	CCCCXIX. DCCCCIIII
					LIX. XLIX	CCCCLXXII. CCCXCII
						DXXXI. CCCCXLI

Boethius: number grids



× (5/4) 1024

× (5/4) 1280

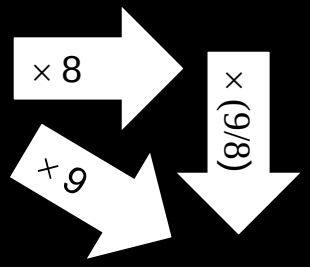
× (5/4) 1600

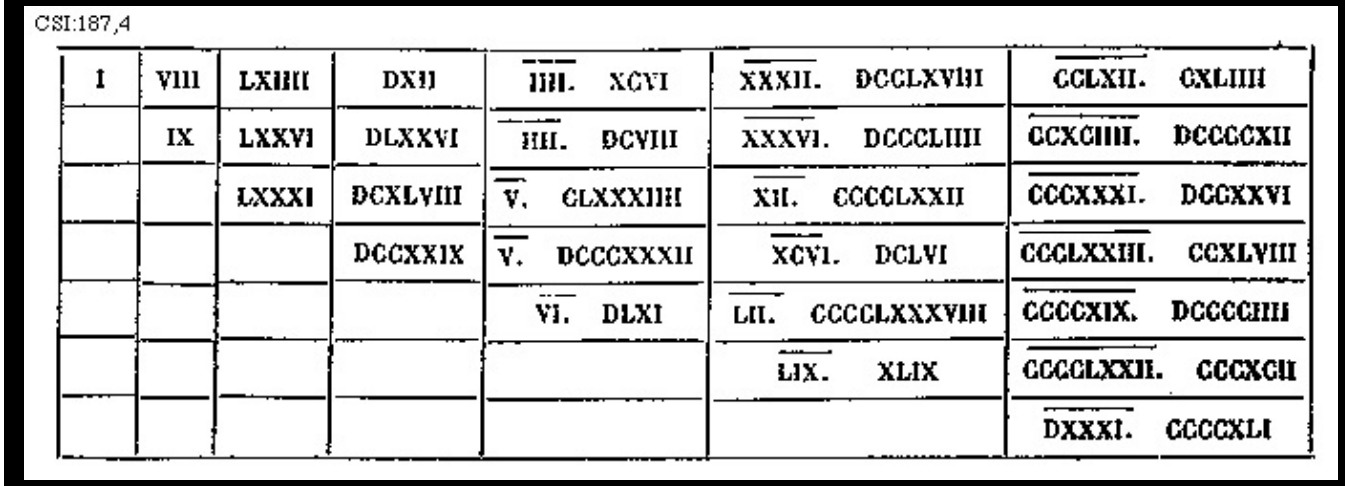
× (5/4) 2000

× (5/4) 2500

× (5/4) 3125

Boethius: number grids



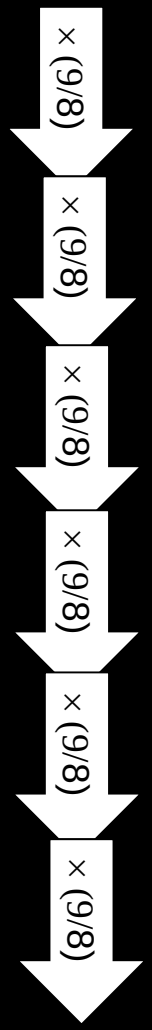


CSI:187,4

I	VIII	LXIIII	DXII	IIII. XCVI	XXXII. DCCLXVIII	CCLXII. CXLIIII
	IX	LXXVI	DLXXVI	III. DCVIII	XXXVI. DCCCLIIII	CCXCIIII. DCCCCXII
		LXXXI	DCXLVIII	V. CLXXXIIII	XII. CCCCLXXII	CCCXXXI. DCCXXVI
			DCCXXIX	V. DCCCXXXII	XCVI. DCLVI	CCCLXXIII. CCXLVIII
				VI. DLXI	LII. CCCCLXXXVIII	CCCCXIX. DCCCCIIII
					LIX. XLIX	CCCCLXXII. CCCXCII
						DXXXI. CCCCXLI

Staatsbibliothek Bamberg Msc.Class. 9 (ca. 1000)

Pythagorean comma:
531,441 : 524,288



262,144

294,912

331,776

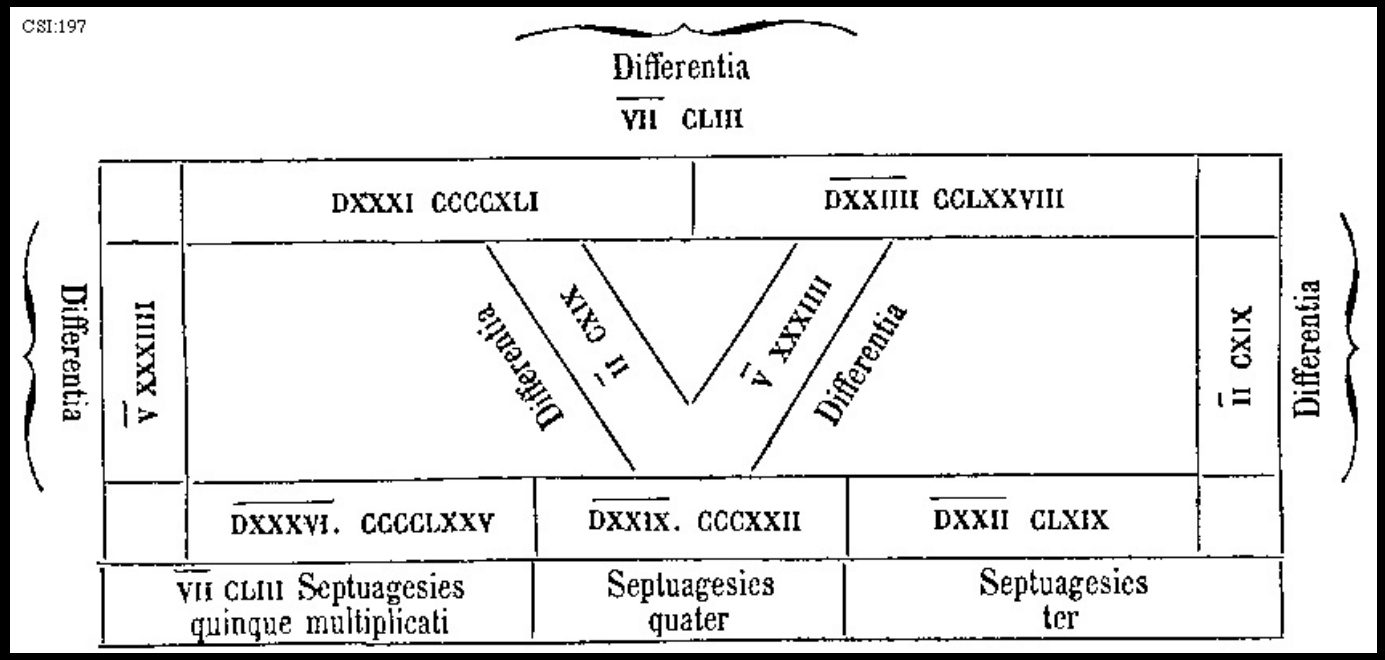
373,248

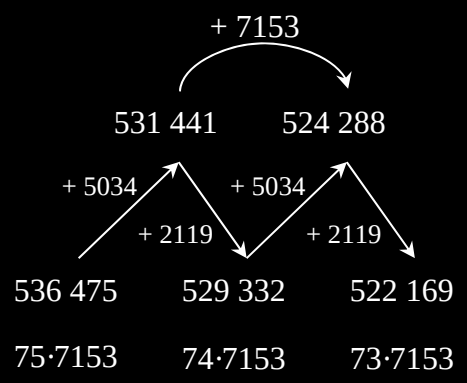
419,904

472,392

531,441

Pythagorean comma (Odington)





$$\frac{75}{74} < c_P < \frac{74}{73}$$

$3c_P \ll s < 4c_P$ (s = 256/243)

$41c_P \ll \text{octave} < 53c_P$

Pythagorean comma (Stifel)

Eſt autem Comma differentia qua ſemitonium maius ſupe rat ſemitonium minus.

Poſſes iam etiam facere progreſſionem Reciſorum & Com matum. Sunt autem triplicia reciſa: quædam ſunt prima, quædam ſunt ſecunda, & quædam tertia. Eſt autem reciſum primũ differentia ſemitonij minoris ſupra comma, ut

134217728.
129140163.

Et reciſum ſecundum eſt differentia, qua reciſum primum ſupe rat comma, ut 1899994288579692 8.
1852997966631841.

Reciſum tertium, eſt differentia, qua reciſum ſecundum ſuperat comma, ut 9961172557086997872 64.
9847590923847922128 81.

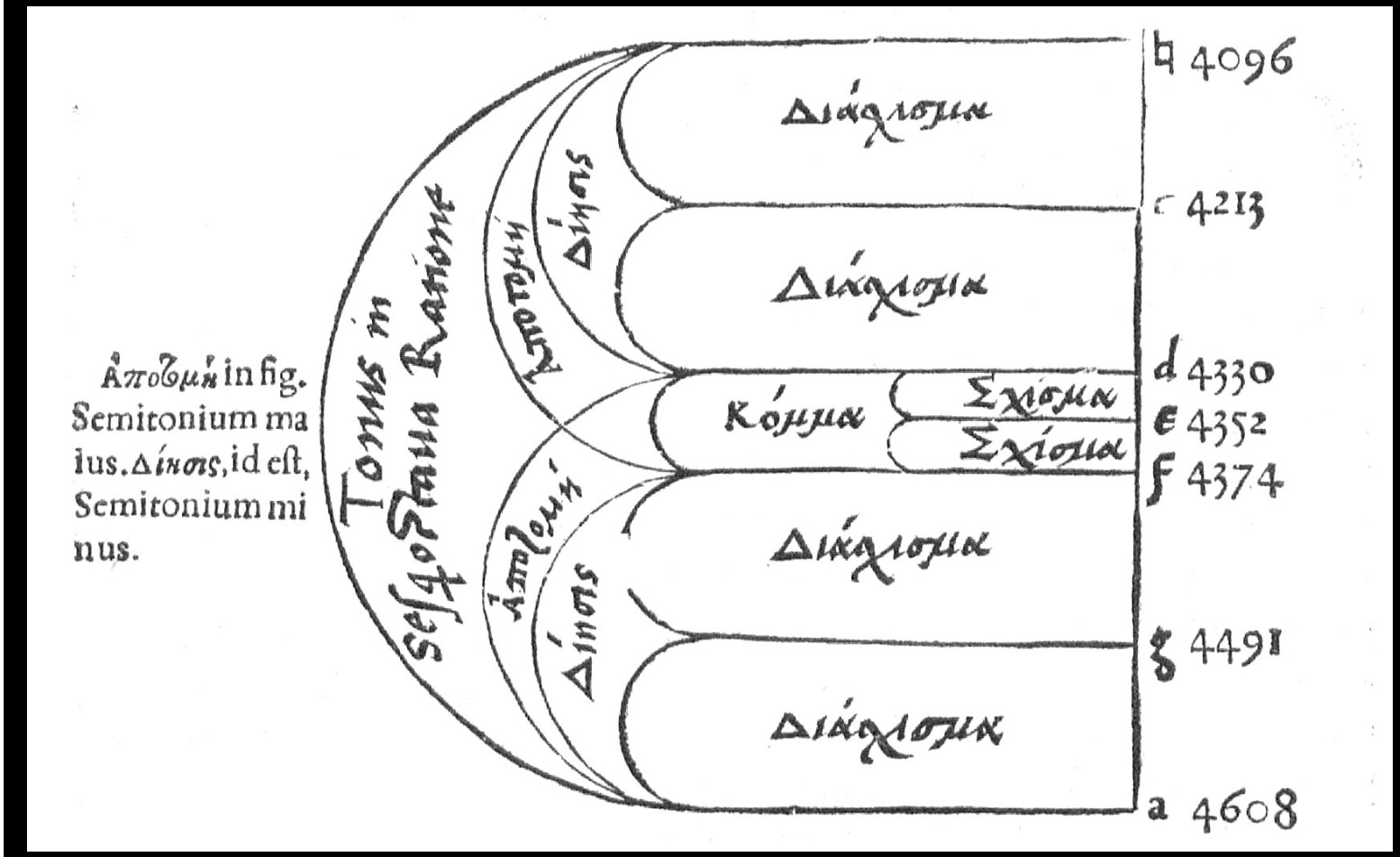
$s - 1c_P$

$s - 2c_P$

$s - 3c_P$

$s - 3c_P < c_P$

Glarean's comma



+ 234

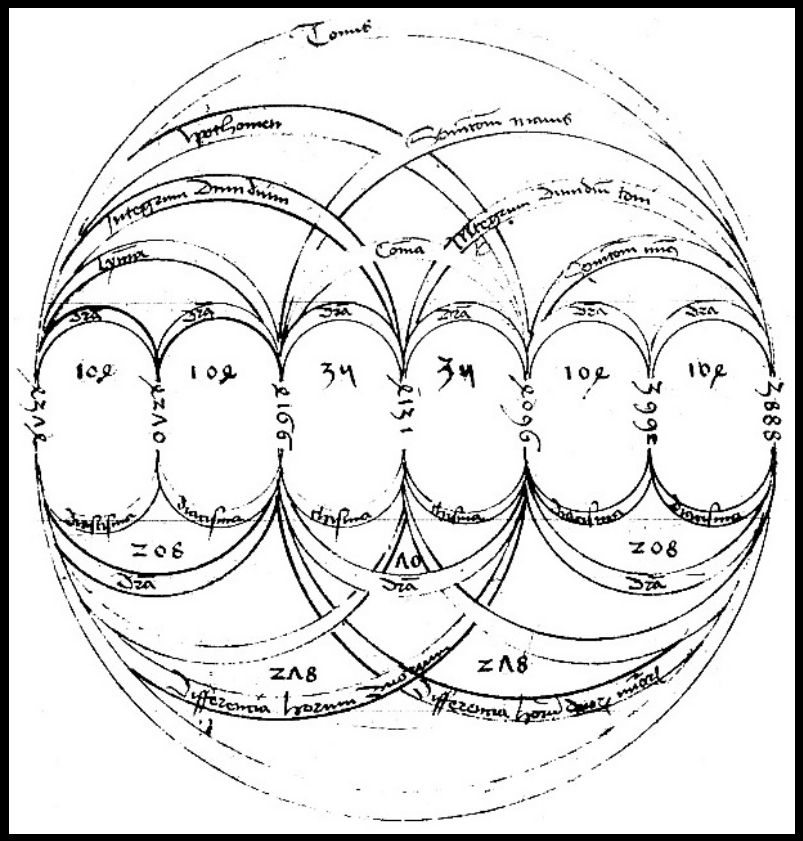
+ 44

+ 234

4608 : 4374 = 256 : 243

4374 : 4330 = 1.0102 < 1.0136 = c_P

Johannes de Muris's comma



4374 : 3888 = 9 : 8

4096 : 3888 = 256 : 234

4166 : 4096 = 1.0171

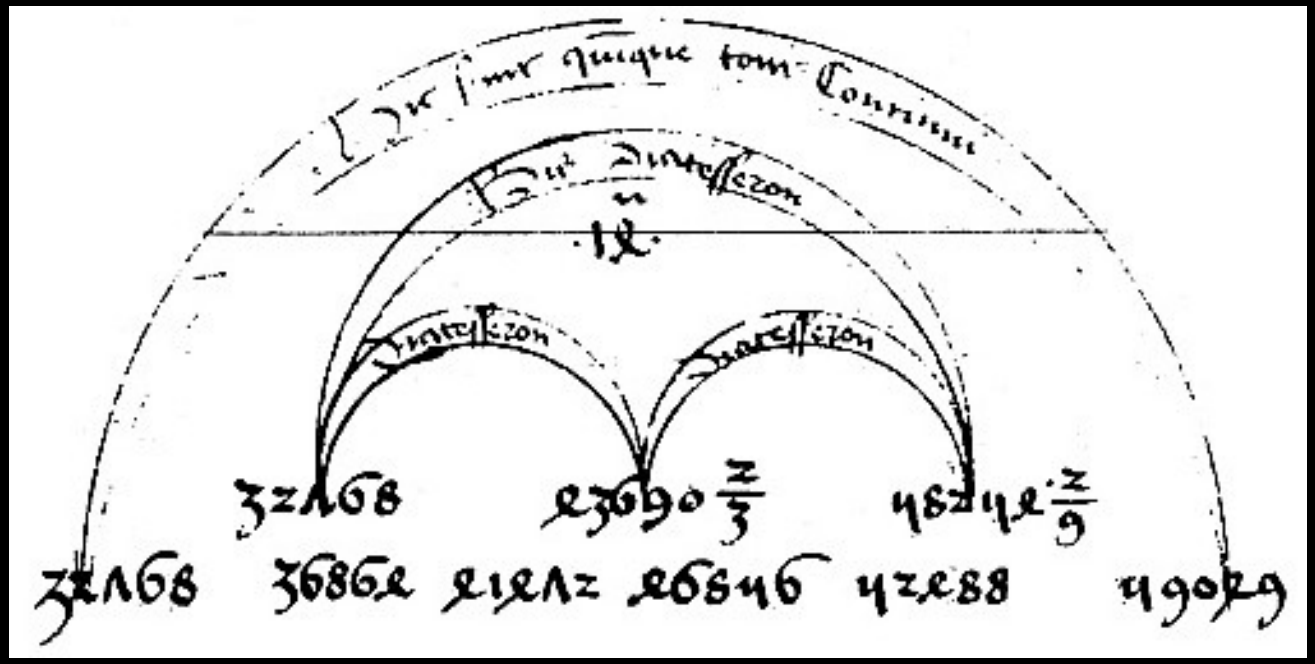
> 1.0136 = c_P

4374 4270 4166 4131 4096 3992 3888

104 104 35 35 104 104

208 70 208

Johannes de Muris: «quinque toni continui»



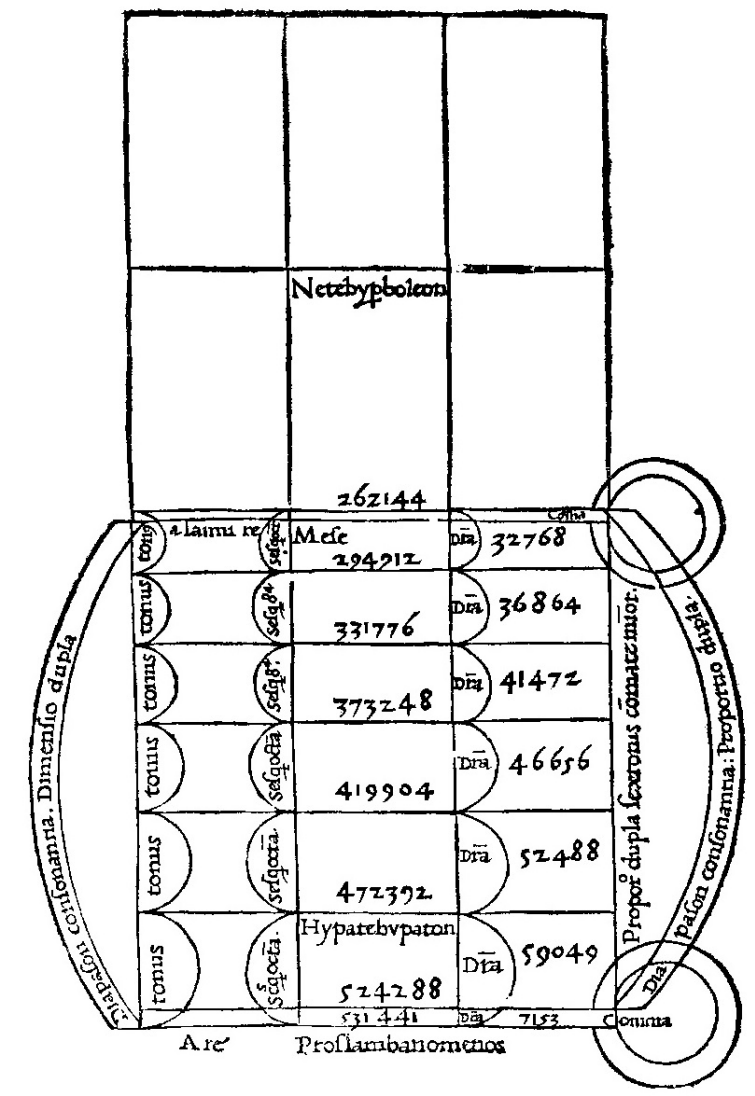
×(4/3) ×(4/3)

32768 43690 2/3 58254 2/9

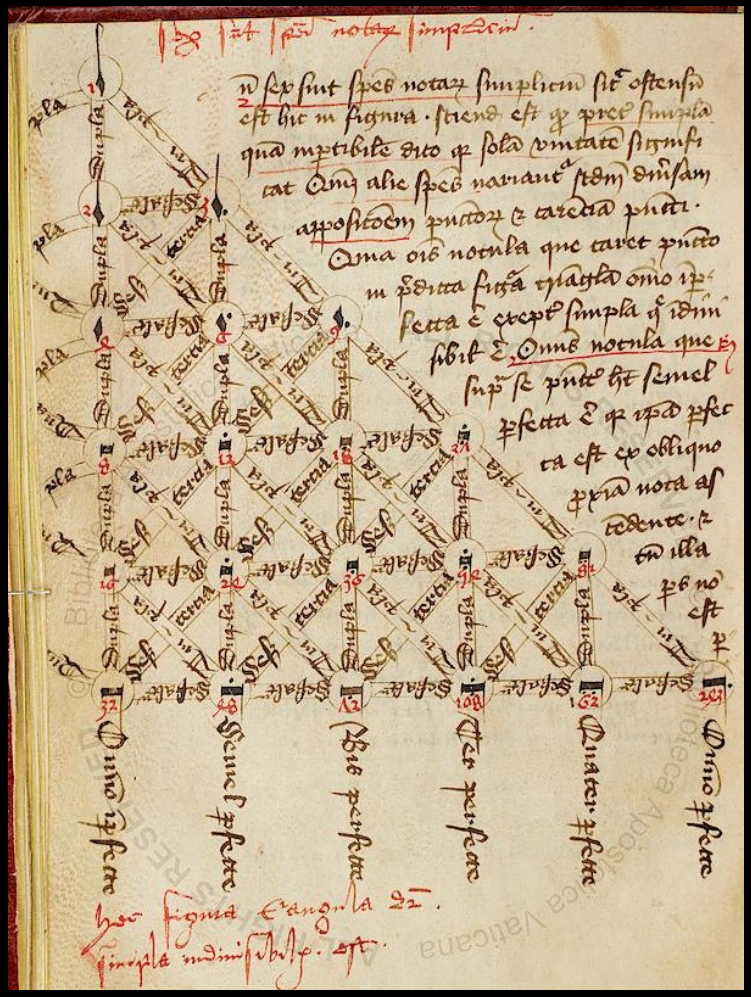
32768 36864 41472 46656 52488 59049

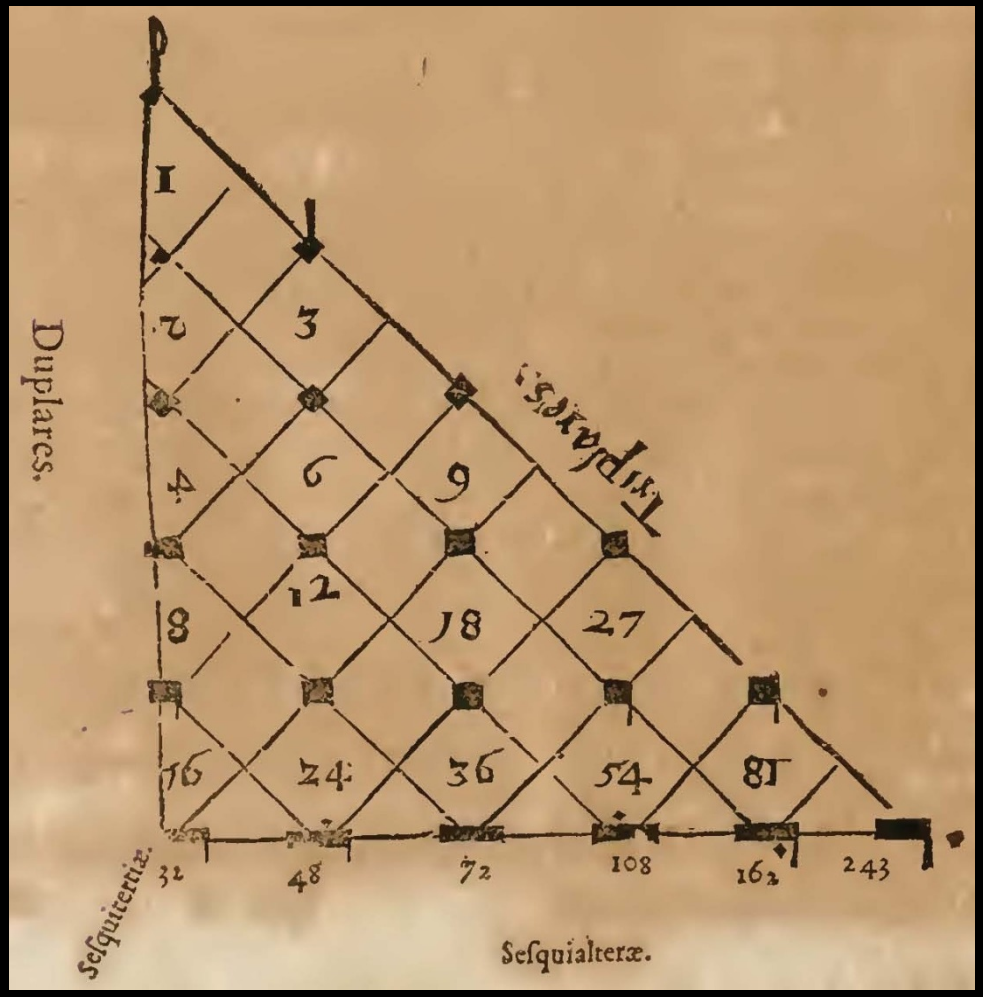
×(9/8) ×(9/8) ×(9/8) ×(9/8) ×(9/8)

Franchino Gaffurio: «Proportio dupla sextonis commate minor»

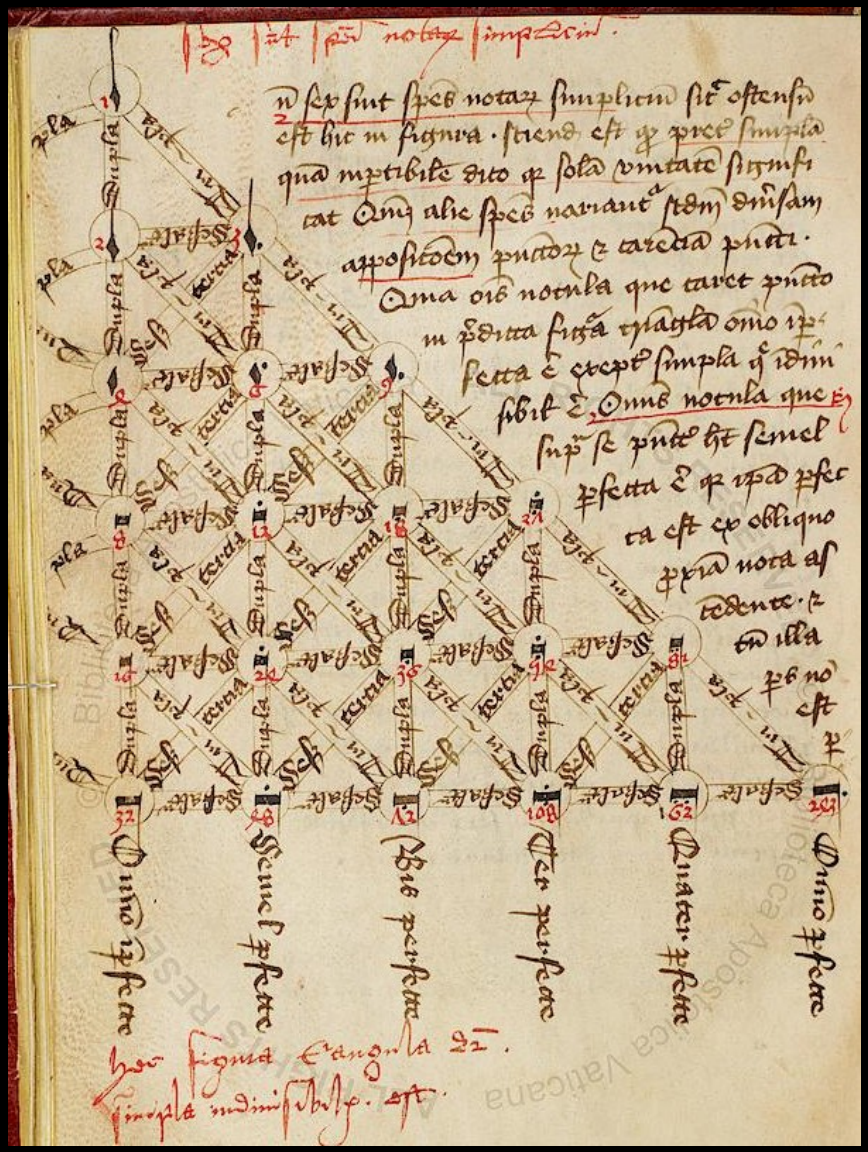


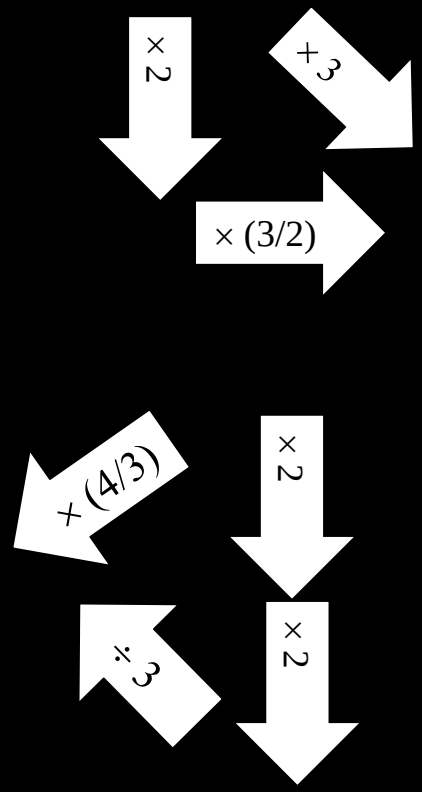
Torkesey/Fludd: number grids



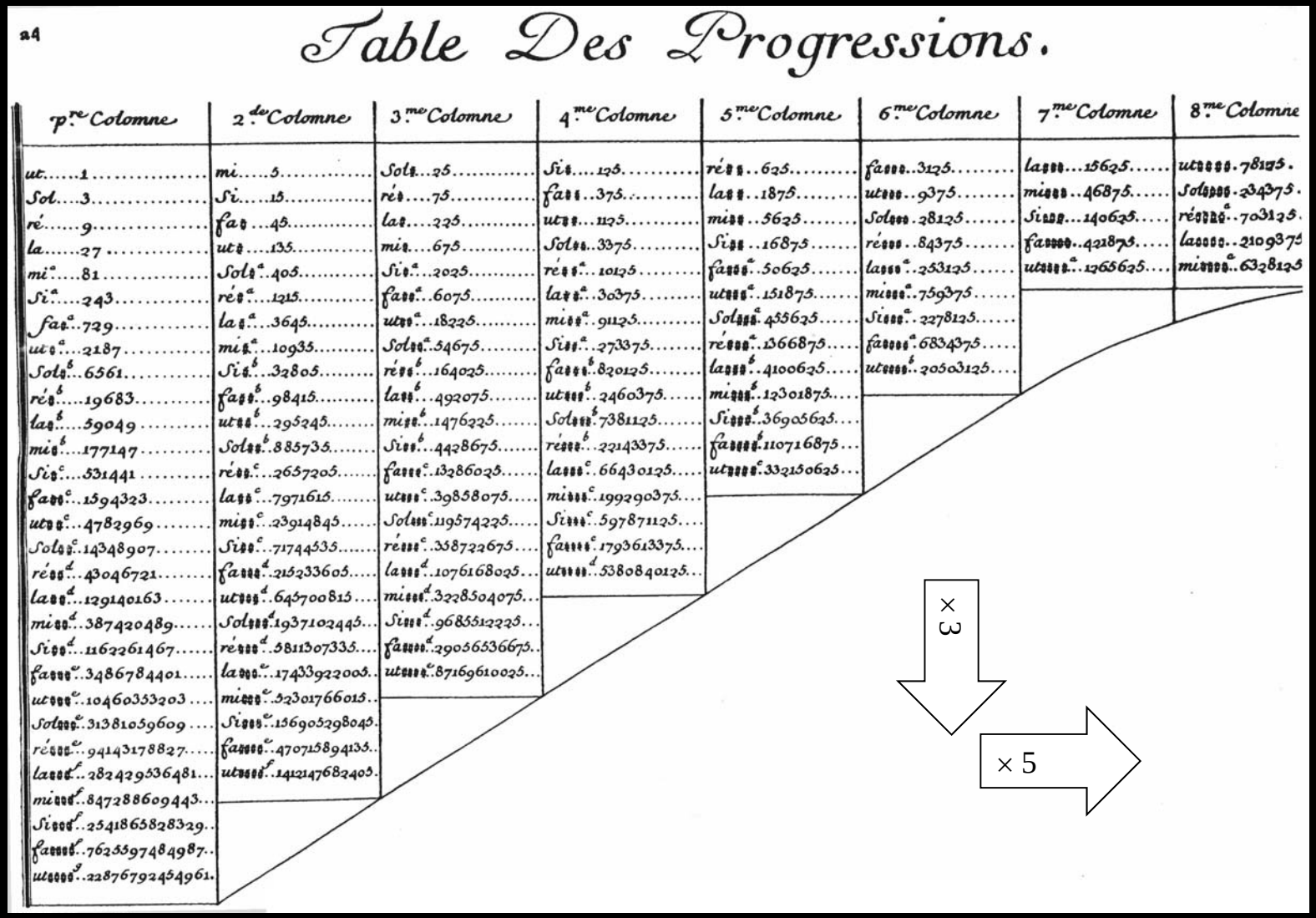


Torkesey: number grid





Rameau (1726): pitch grid



a4

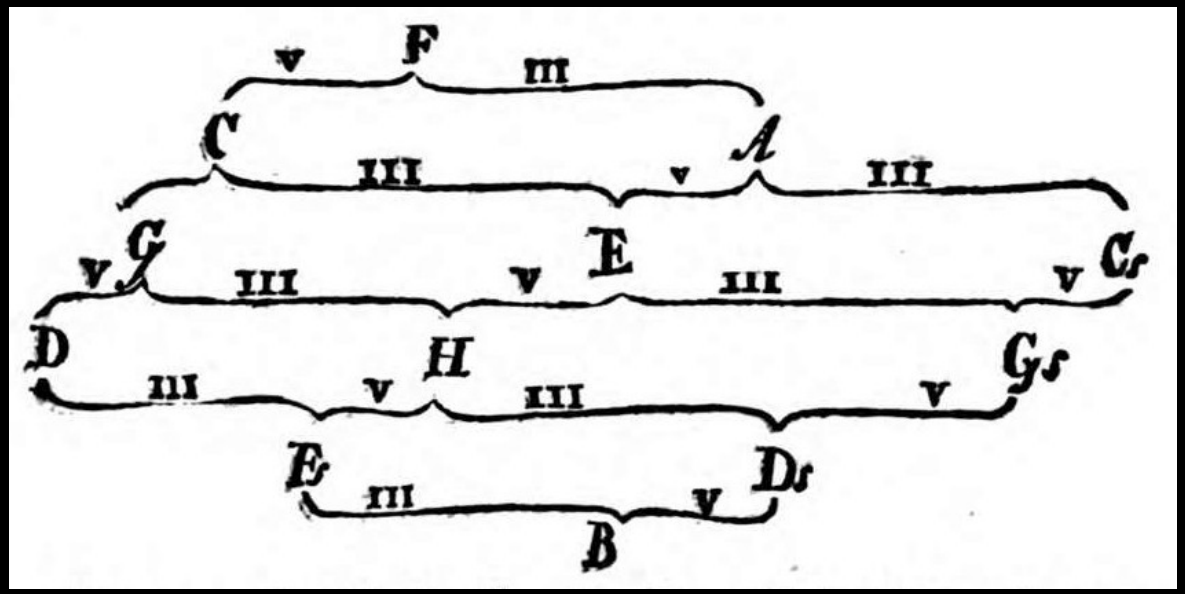
Table Des Progressions.

1re Colomne	2de Colomne	3me Colomne	4me Colomne	5me Colomne	6me Colomne	7me Colomne	8me Colomne
ut.....1	mi.....5	Sol.....25	Si.....125	ré.....625	fa.....3125	la.....15625	ut.....78125
Sol.....3	Si.....15	ré.....75	fa.....375	la.....1875	ut.....9375	mi.....46875	Sol.....234375
ré.....9	fa.....45	la.....225	ut.....1125	mi.....5625	Sol.....28125	Si.....140625	ré^{a}.....703125
la.....27	ut.....135	mi.....675	Sol.....3375	Si.....16875	ré.....84375	fa.....421875	la.....2109375
mi^{a}.....81	Sol^{a}.....405	Si^{a}.....2025	ré^{a}.....10125	fa^{a}.....50625	la^{a}.....253125	ut^{a}.....1265625	mi^{a}.....6328125
Si^{a}.....243	ré^{a}.....1215	fa^{a}.....6075	la^{a}.....30375	ut^{a}.....151875	mi^{a}.....759375		
fa^{a}.....729	la^{a}.....3645	ut^{a}.....18225	mi^{a}.....91125	Sol^{a}.....455625	Si^{a}.....2278125		
ut^{a}.....2187	mi^{a}.....10935	Sol^{a}.....54675	Si^{a}.....273375	ré^{a}.....1366875	fa^{a}.....6834375		
Sol^{b}.....6561	Si^{b}.....32805	ré^{b}.....164025	fa^{b}.....820125	la^{b}.....4100625	ut^{b}.....20503125		
ré^{b}.....19683	fa^{b}.....98415	la^{b}.....492075	ut^{b}.....2460375	mi^{b}.....12301875			
la^{b}.....59049	ut^{b}.....295245	mi^{b}.....1476225	Sol^{b}.....7381125	Si^{b}.....36905625			
mi^{b}.....177147	Sol^{b}.....885735	Si^{b}.....4428675	ré^{b}.....22143375	fa^{b}.....110716875			
Si^{c}.....531441	ré^{c}.....2657205	fa^{c}.....13286025	la^{c}.....66430125	ut^{c}.....332150625			
fa^{c}.....1594323	la^{c}.....7971615	ut^{c}.....39858075	mi^{c}.....199290375				
ut^{c}.....4782969	mi^{c}.....23914845	Sol^{c}.....119574225	Si^{c}.....597871125				
Sol^{c}.....14348907	Si^{c}.....71744535	ré^{c}.....358722675	fa^{c}.....1793613375				
ré^{d}.....43046721	fa^{d}.....215233605	la^{d}.....1076168025	ut^{d}.....5380840125				
la^{d}.....129140163	ut^{d}.....645700815	mi^{d}.....3228504075					
mi^{d}.....387420489	Sol^{d}.....1937102445	Si^{d}.....9685512225					
Si^{d}.....1162261467	ré^{d}.....5811307335	fa^{d}.....29056536675					
fa^{e}.....3486784401	la^{e}.....17433922005	ut^{e}.....87169610025					
ut^{e}.....10460353203	mi^{e}.....52301766015						
Sol^{e}.....31381059609	Si^{e}.....156905298045						
ré^{e}.....94143178827	fa^{e}.....470715894135						
la^{f}.....282429536481	ut^{f}.....1412147682405						
mi^{f}.....847288609443							
Si^{f}.....2541865828329							
fa^{f}.....7623597484987							
ut^{g}.....22876792454961							

× 3

× 5

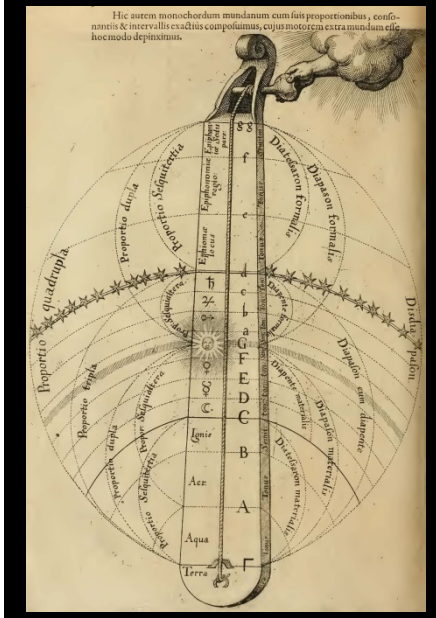
Euler (1739): pitch grid

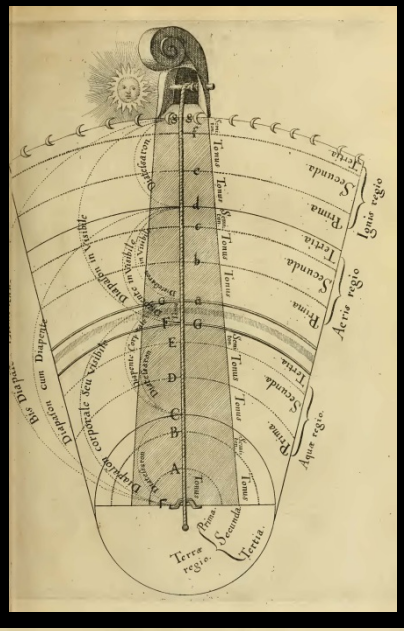


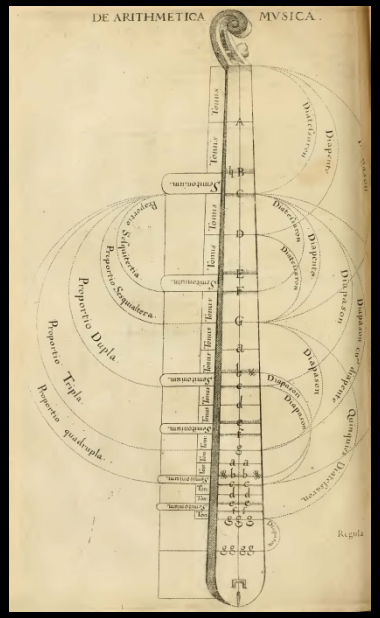
$2^n \cdot 3^3(3)$;	$2^n \cdot 3^3(3^2)$;	$2^n \cdot 3^3(3^3)$;	$2^n \cdot 3^3(3^4)$;
C	G	D	A
$2^n \cdot 3^3(3 \cdot 5)$;	$2^n \cdot 3^3(3^2 \cdot 5)$;	$2^n \cdot 3^3(3^3 \cdot 5)$;	$2^n \cdot 3^3 \cdot (3^4 \cdot 5)$;
E	H	Fs	Cs
$2^n \cdot 3^3(3 \cdot 5^2)$;	$2^n \cdot 3^3(3^2 \cdot 5^2)$;	$2^n \cdot 3^3(3^3 \cdot 5^2)$;	$2^n \cdot 3^3(3^4 \cdot 5^2)$
Gs	Ds	B	F

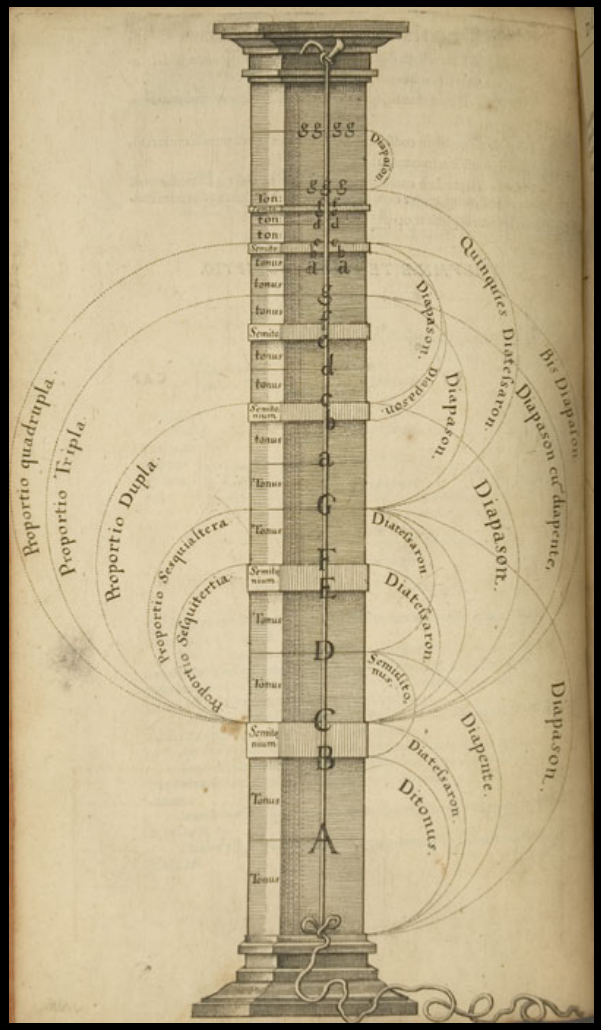
[→]

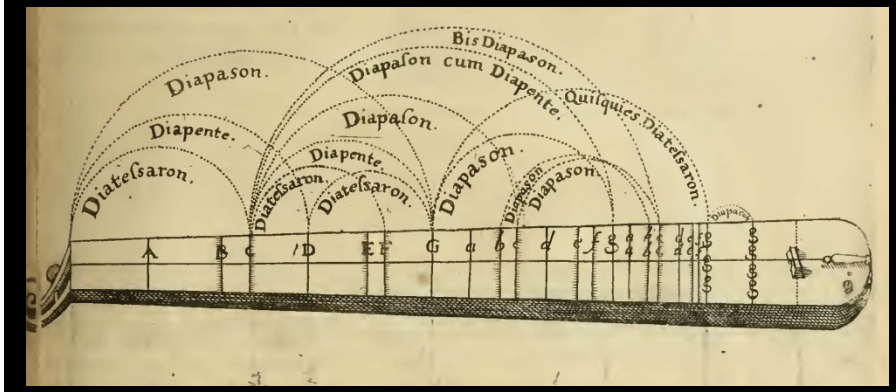
Monochord (Fludd 1617/24)



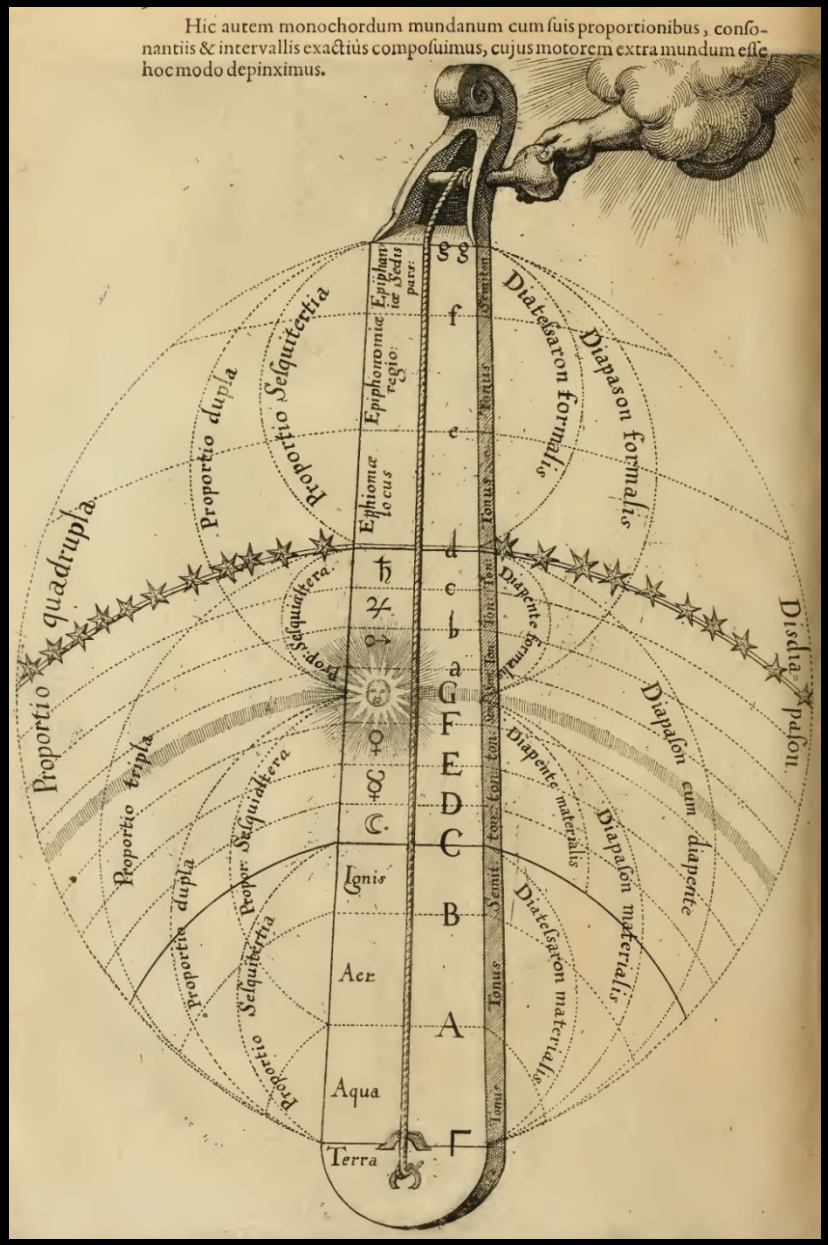


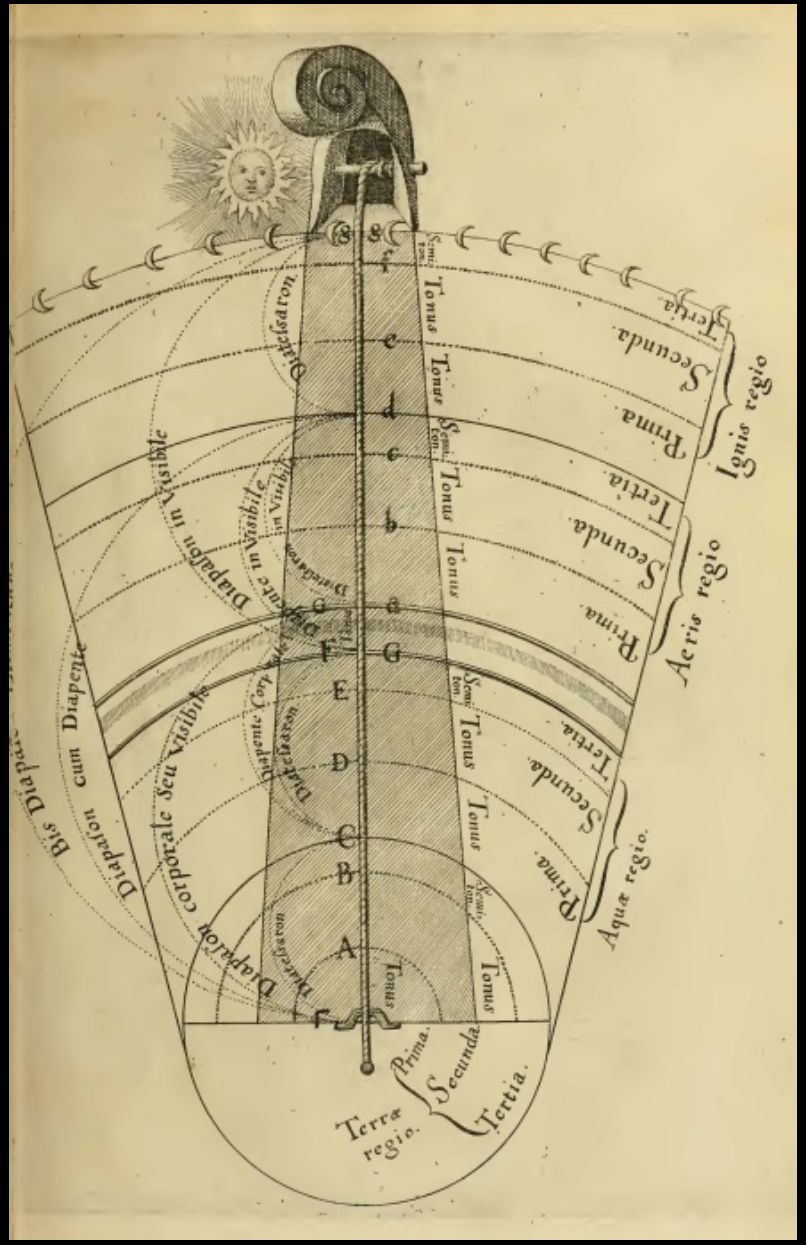




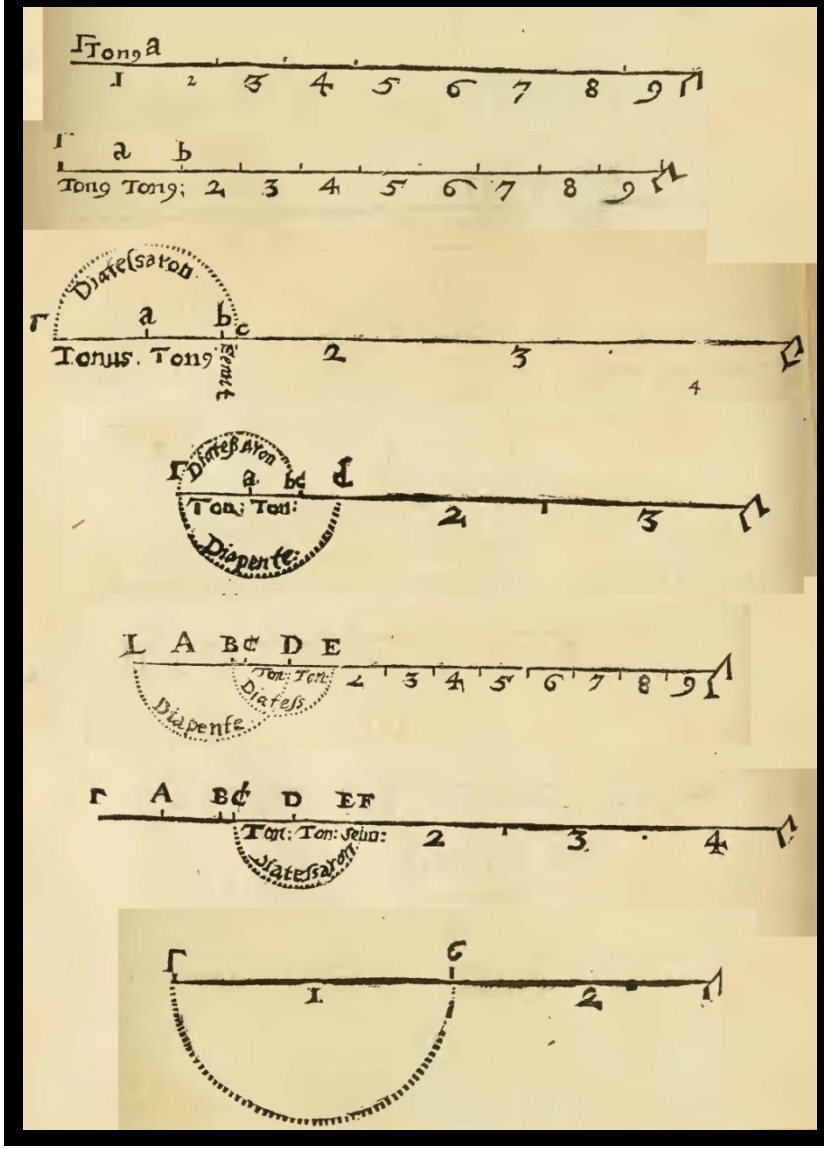


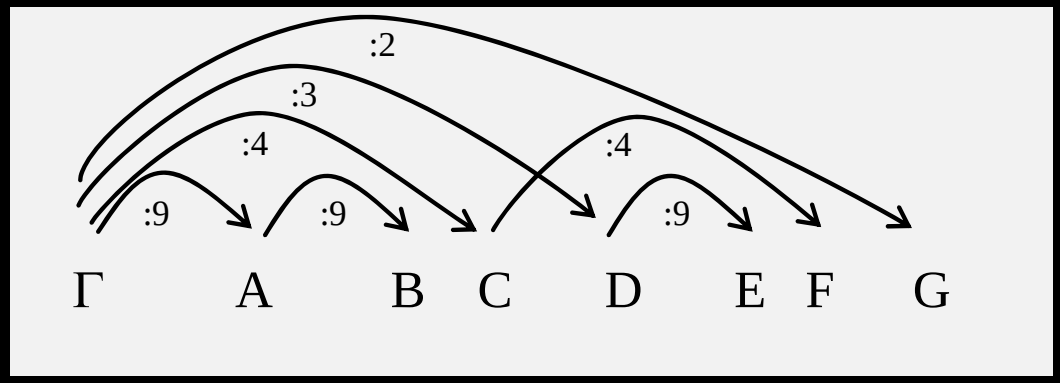
Fludd: Mono-chords





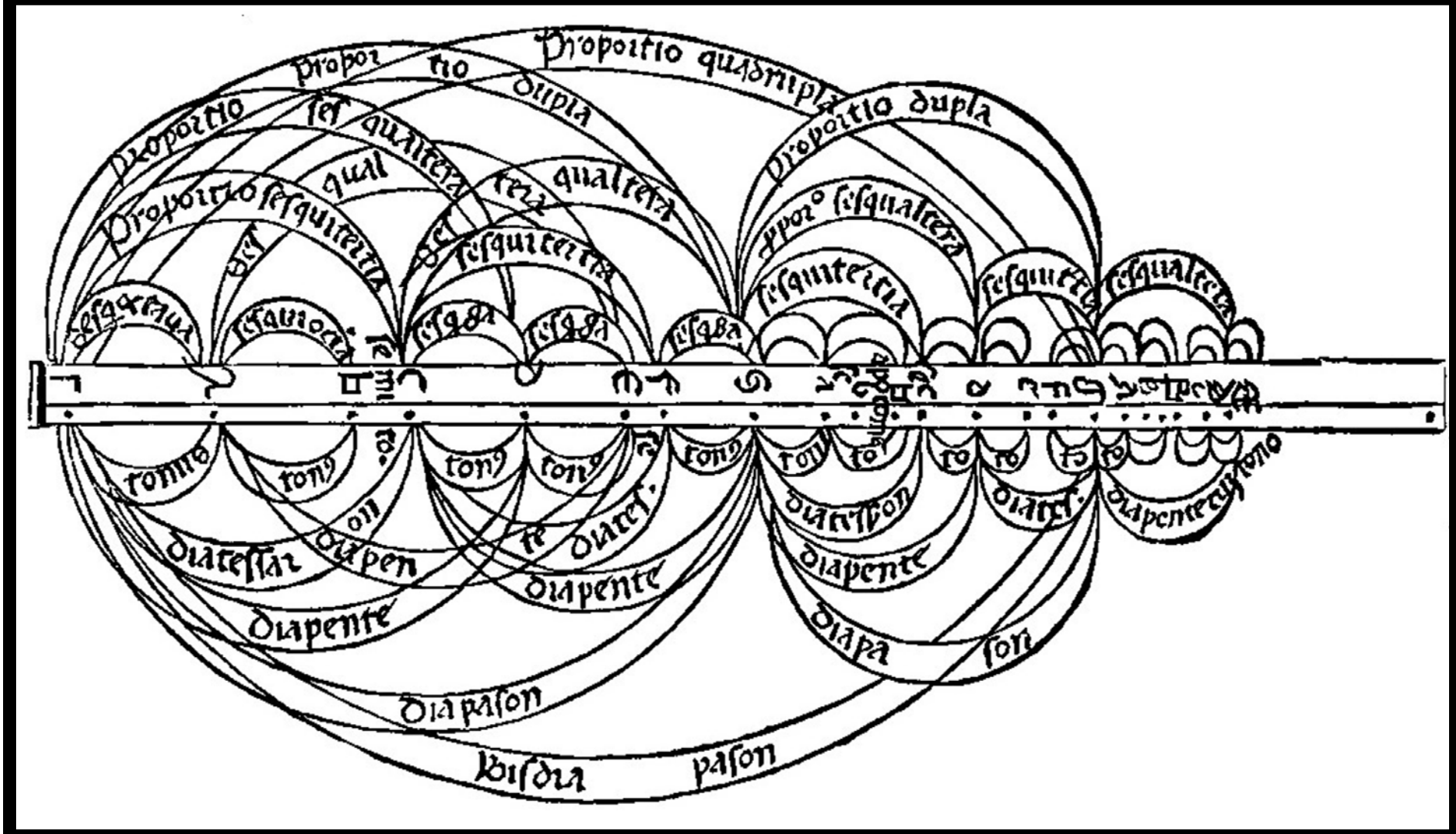
Fludd: monochord tuning



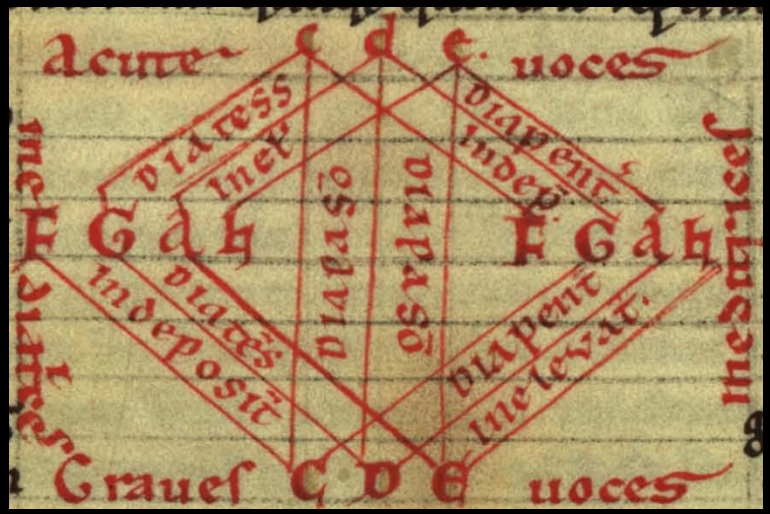


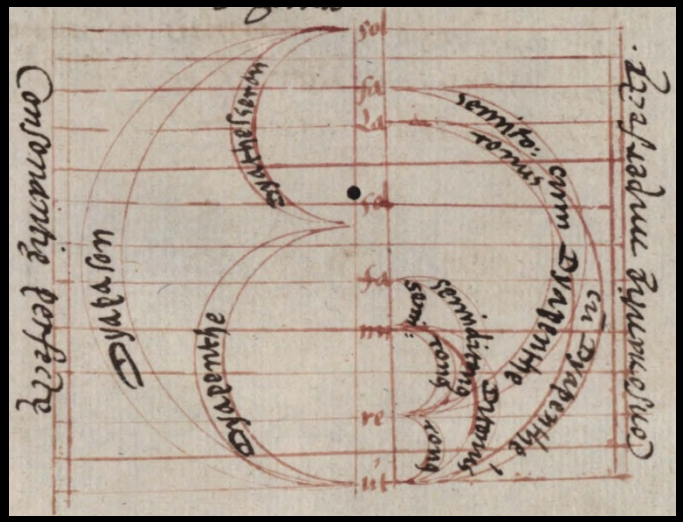
Γ		A		B	C		D		E		F		G
	$\frac{9}{8}$		$\frac{9}{8}$	$\frac{256}{243}$		$\frac{9}{8}$		$\frac{9}{8}$		$\frac{256}{243}$		$\frac{9}{8}$	
		$\frac{4}{3}$					$\frac{4}{3}$					$\frac{9}{8}$	

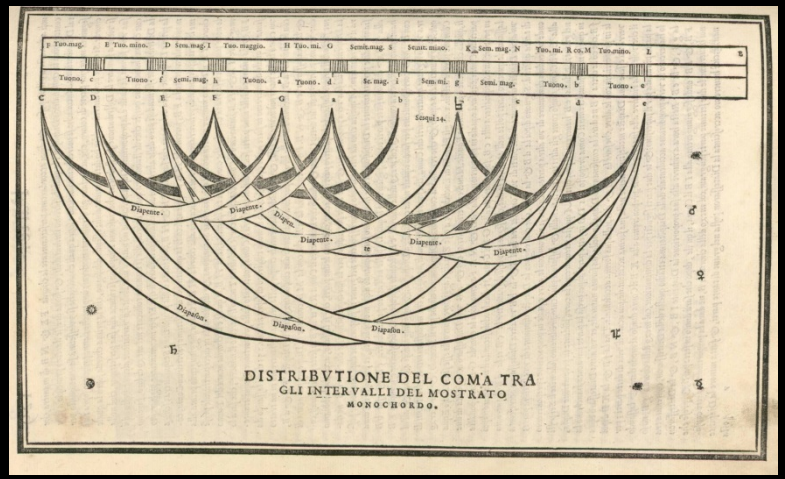
Diatonic scale (Gaffurio 1492)

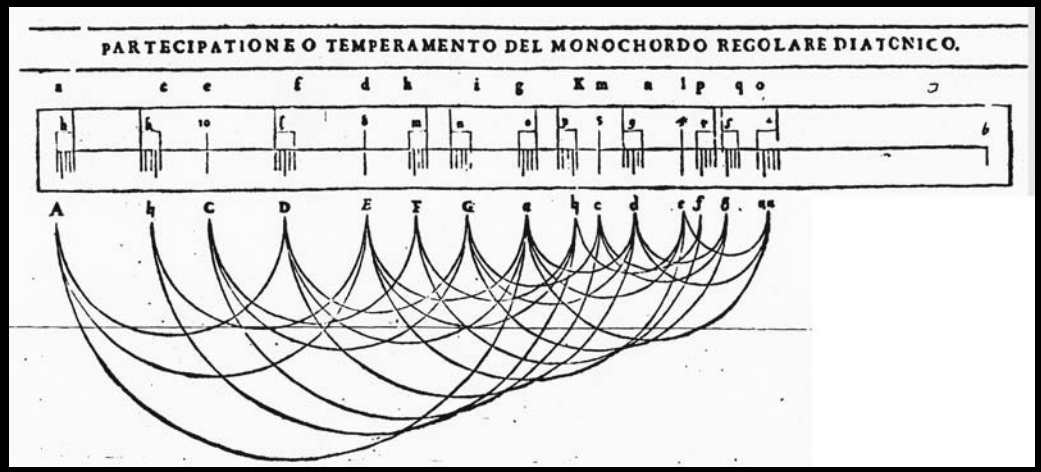

Proportio quadrupla
Proportio dupla
Proportio dupla
Proportio sesquiterta
Proportio sesquiterta
Proportio sesquiterta

Diatonic scale

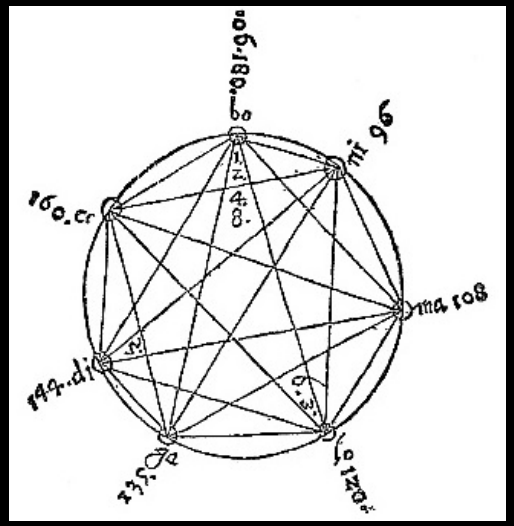


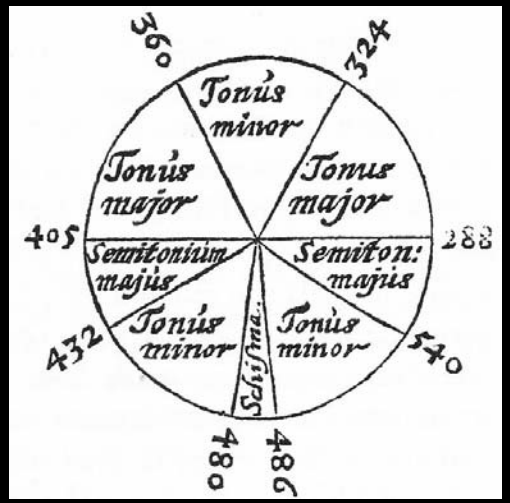


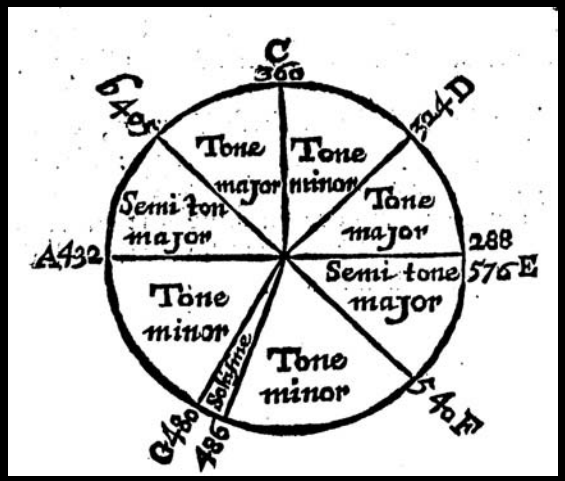


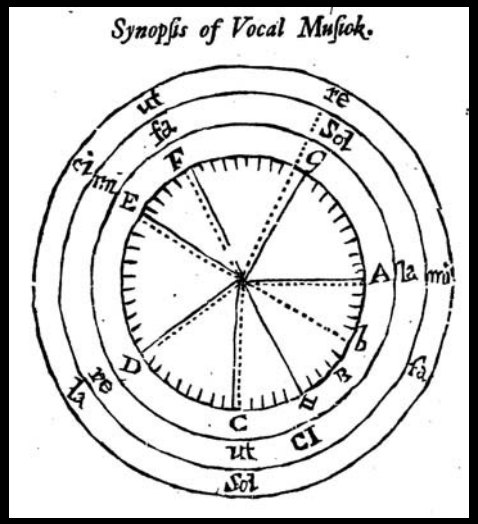


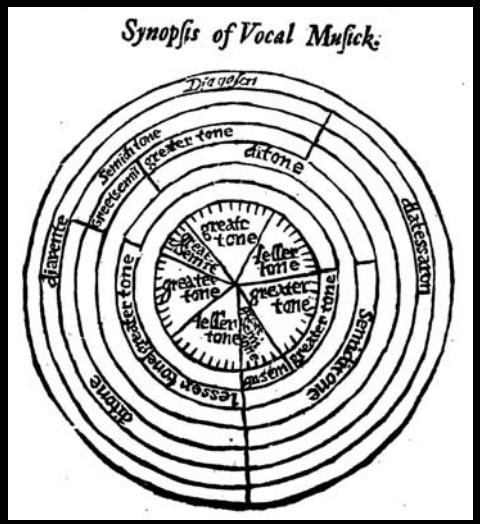
Diatonic scale

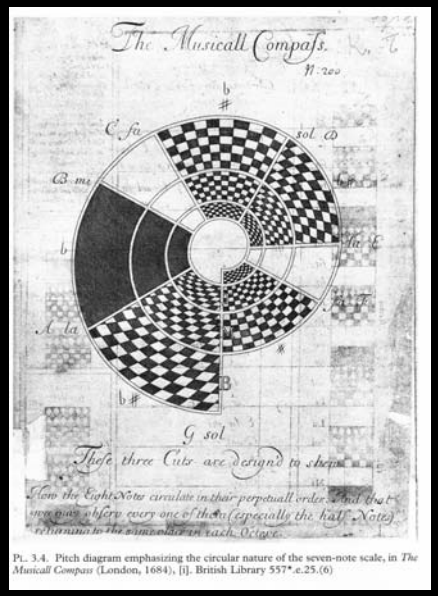






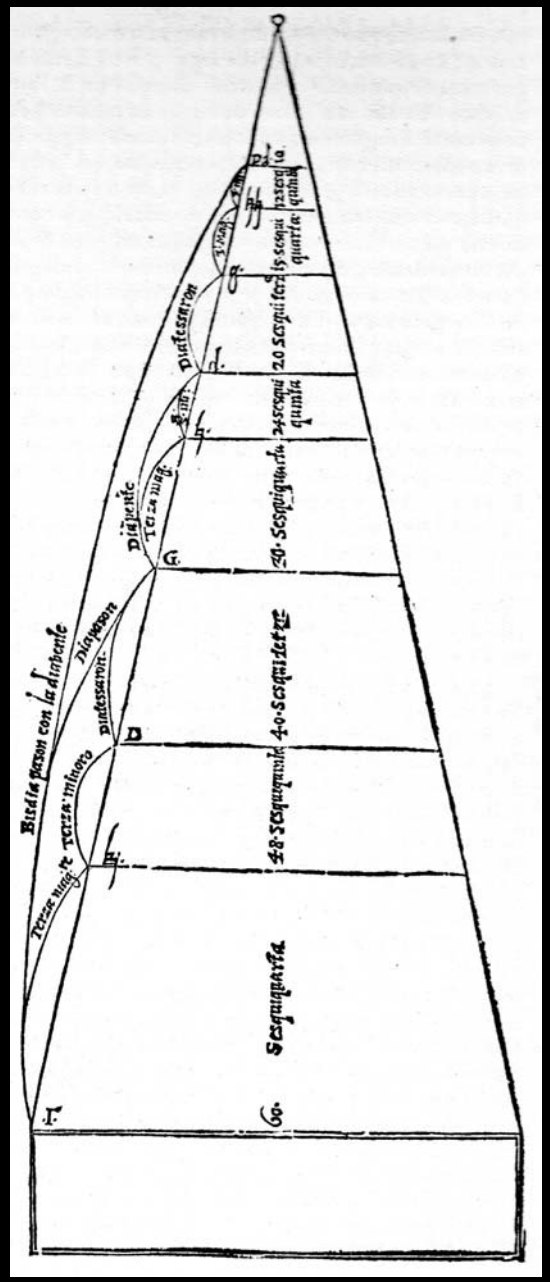






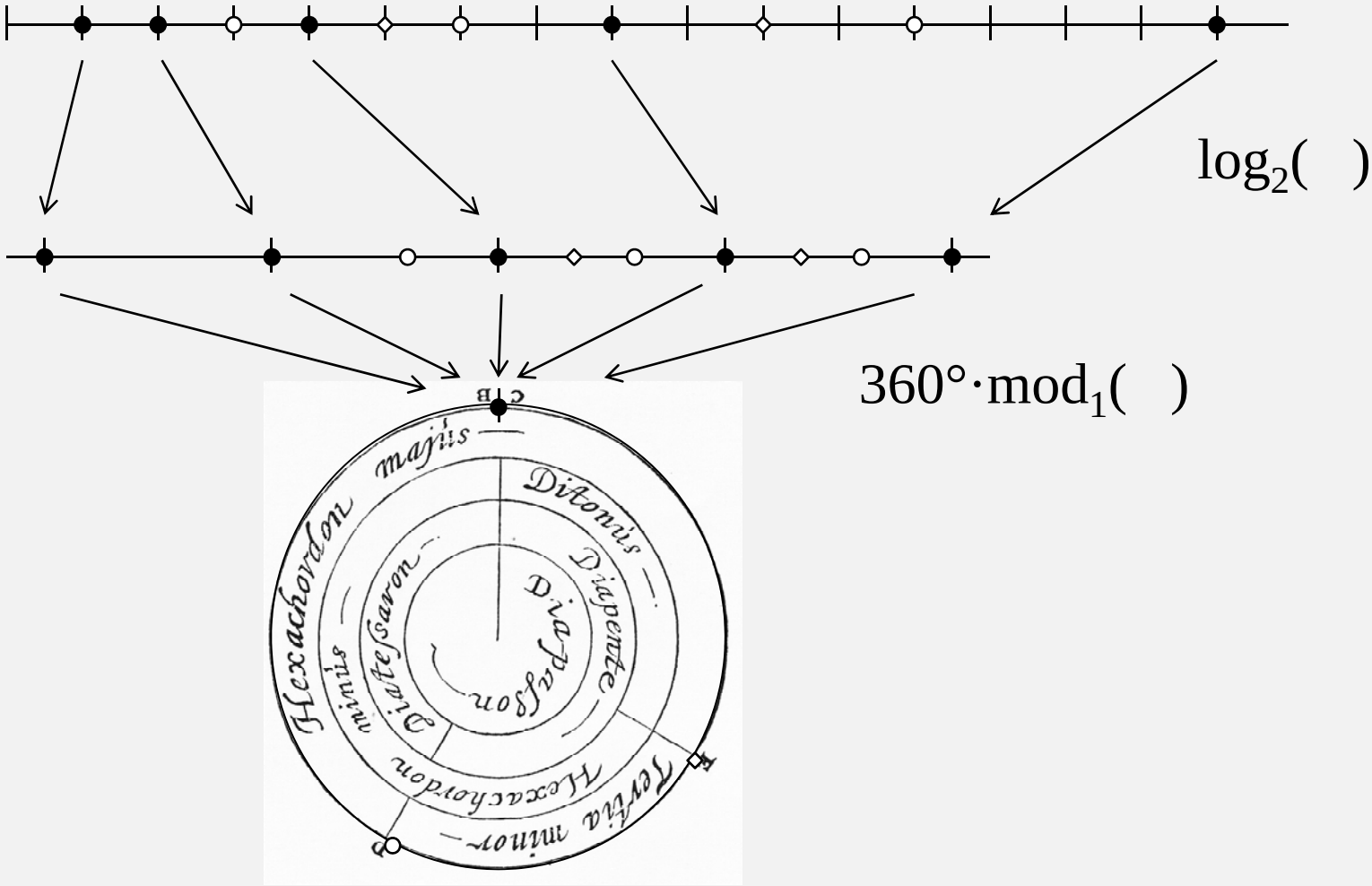
Pl. 3.4. Pitch diagram emphasizing the circular nature of the seven-note scale, in *The Musicall Compass* (London, 1684), [i]. British Library 557*.e.25.(6)

Major triad

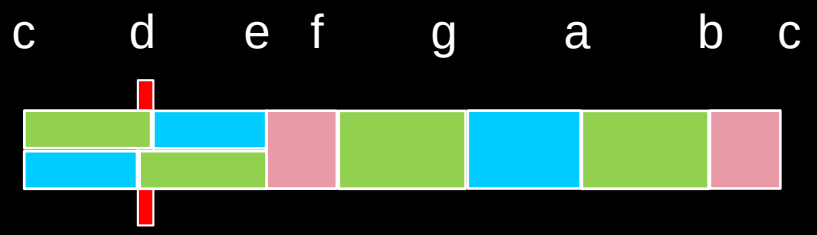


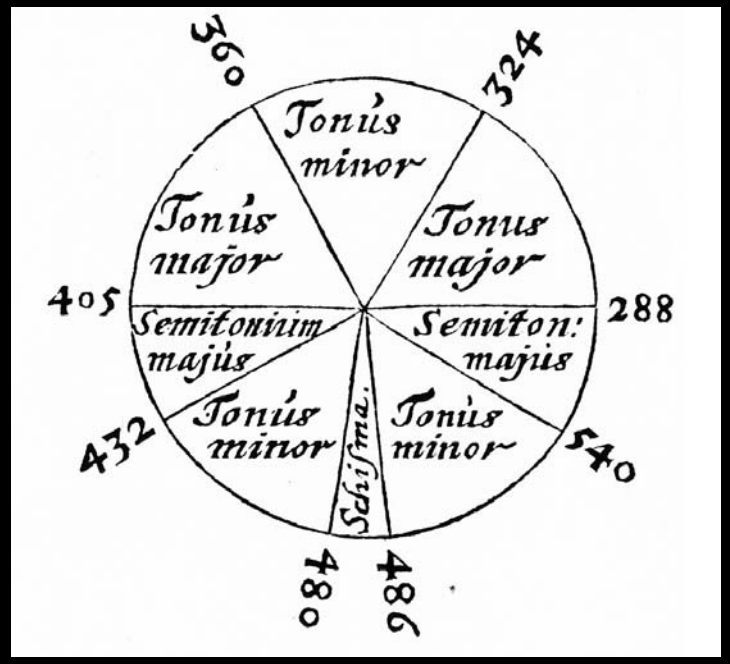
TERTIA MINOR
HEXACHORDON
HEXACHORDON
MINVS
DIATESSARON
DIATESSARON
DIAPENTE
DIAPENTE
DIAPASON
DITONVS
MAIVS

Transformations behind Descartes's diagrams



Descartes' diatonic scale





1650 (1618)

	9:8	major tone
	10:9	minor tone
	16:15	semitone
	81:80	syntonic comma: «Schisma»

[→]

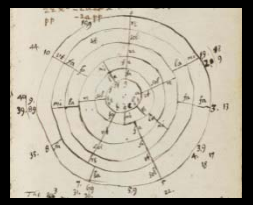
Newton’s diatonic scales

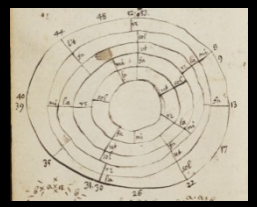
ut re mi fa sol la fa ut

g a b c d e f g

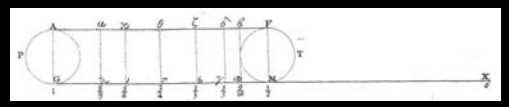
g a bb c d e f g

sol la fa sol la mi fa sol





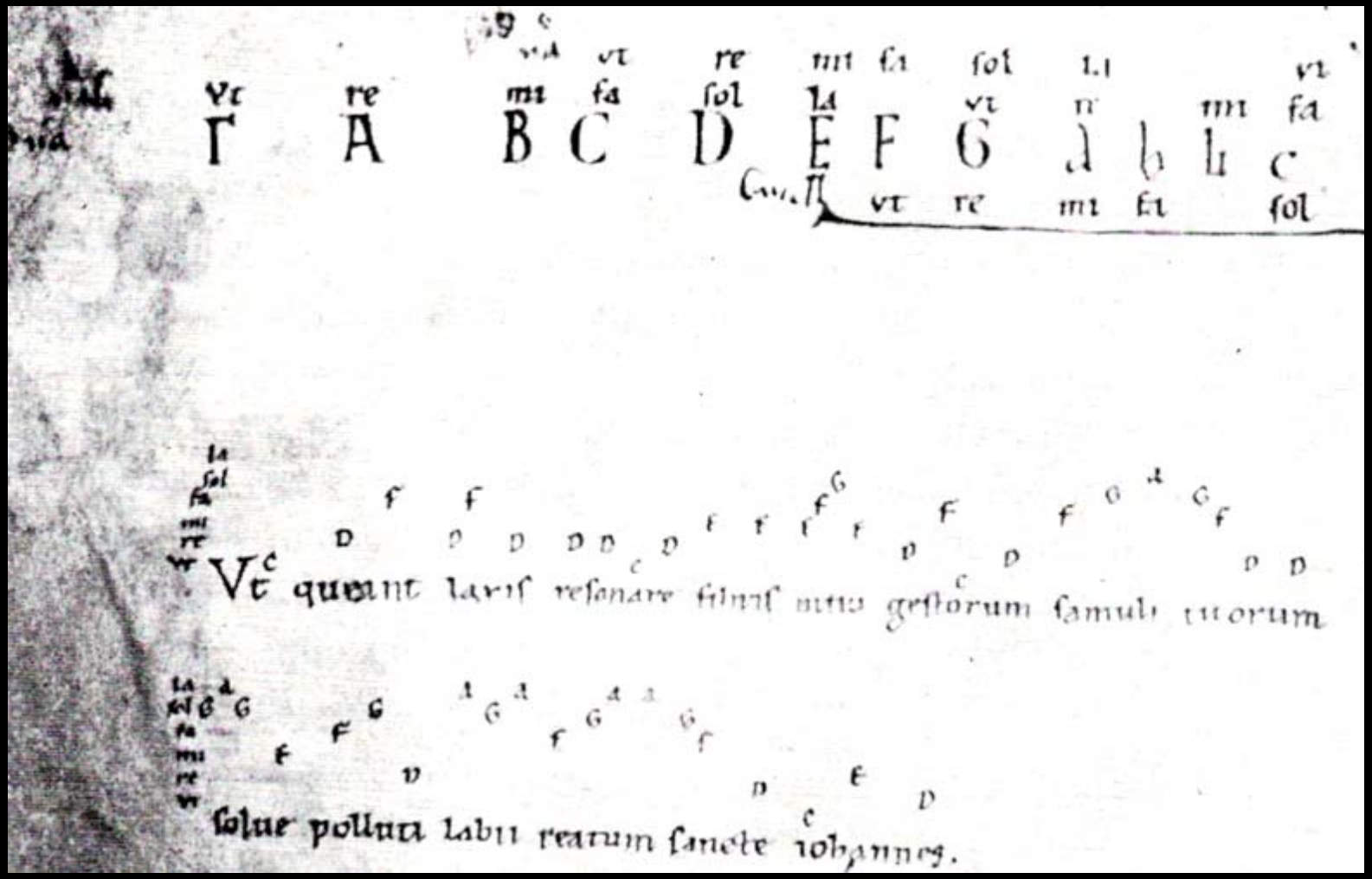
1665



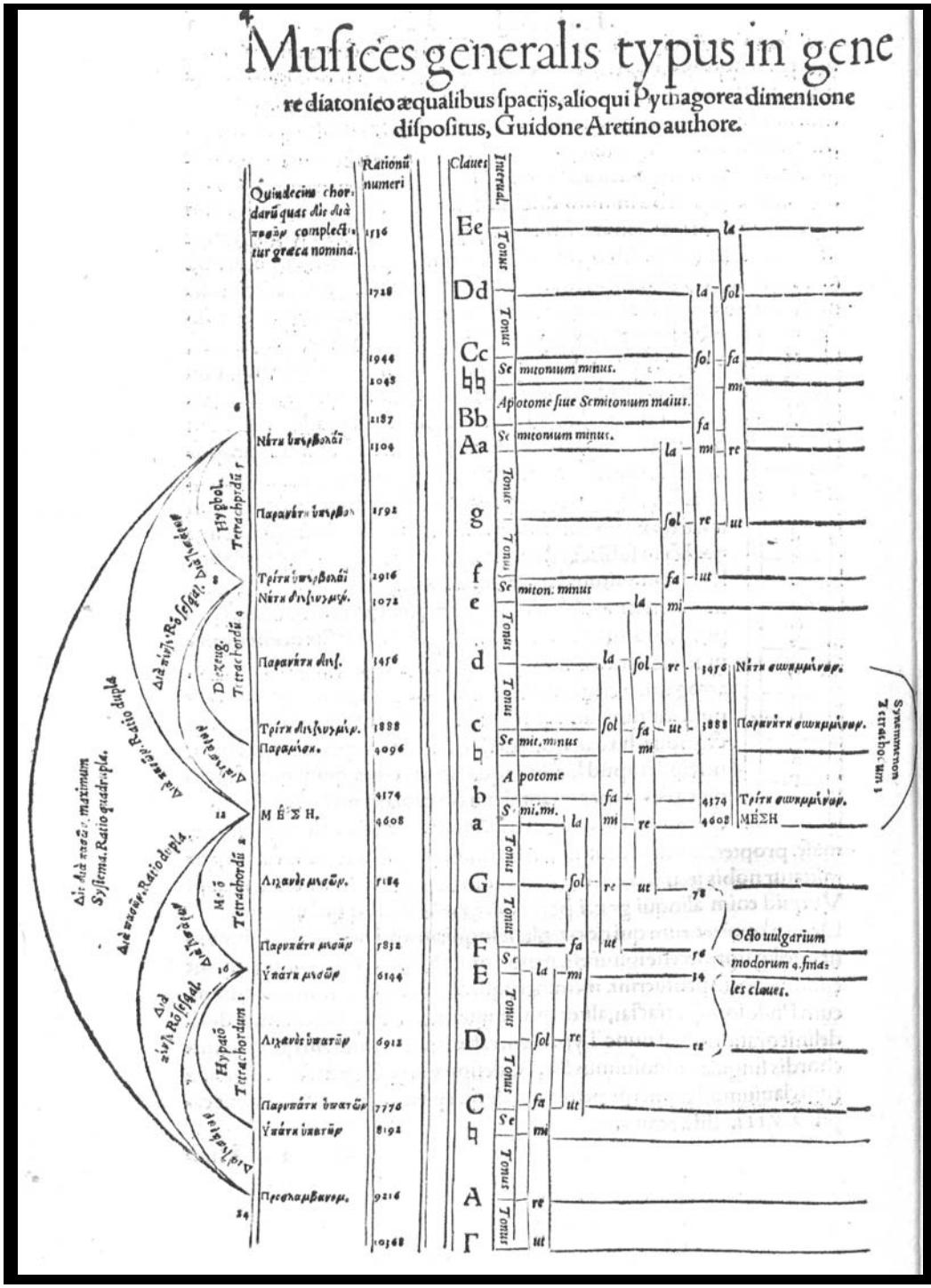
1675/1704

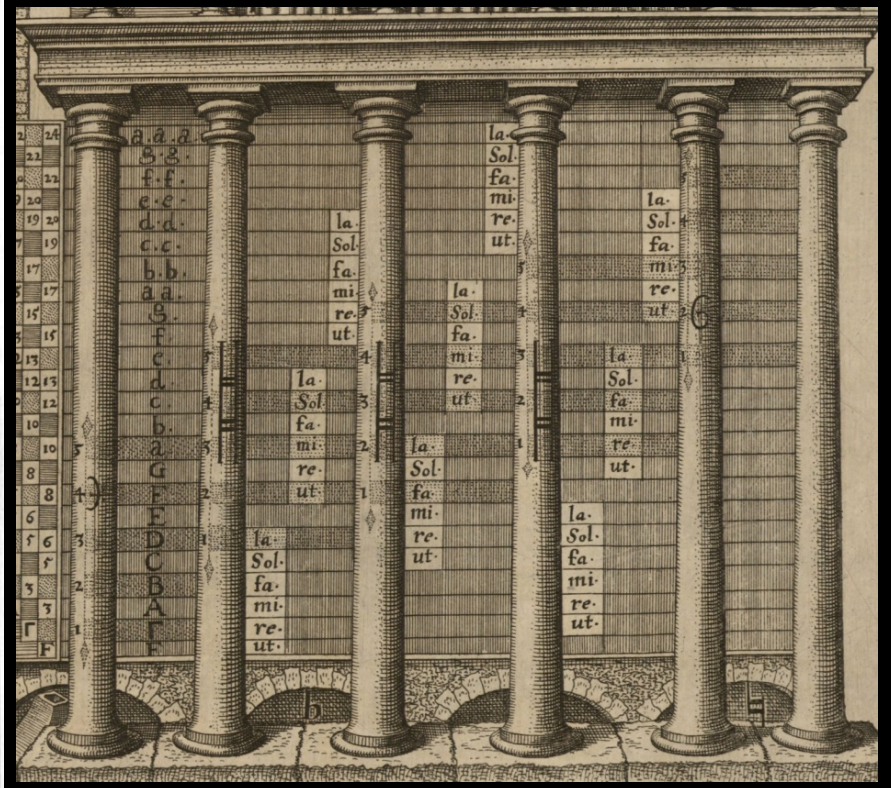
	major tone	9 : 8	9 units (53-tet)
	minor tone	10 : 9	8 units (53-tet)
	semitone	16 :15	5 units (53-tet)

Hexachords (11th c.)



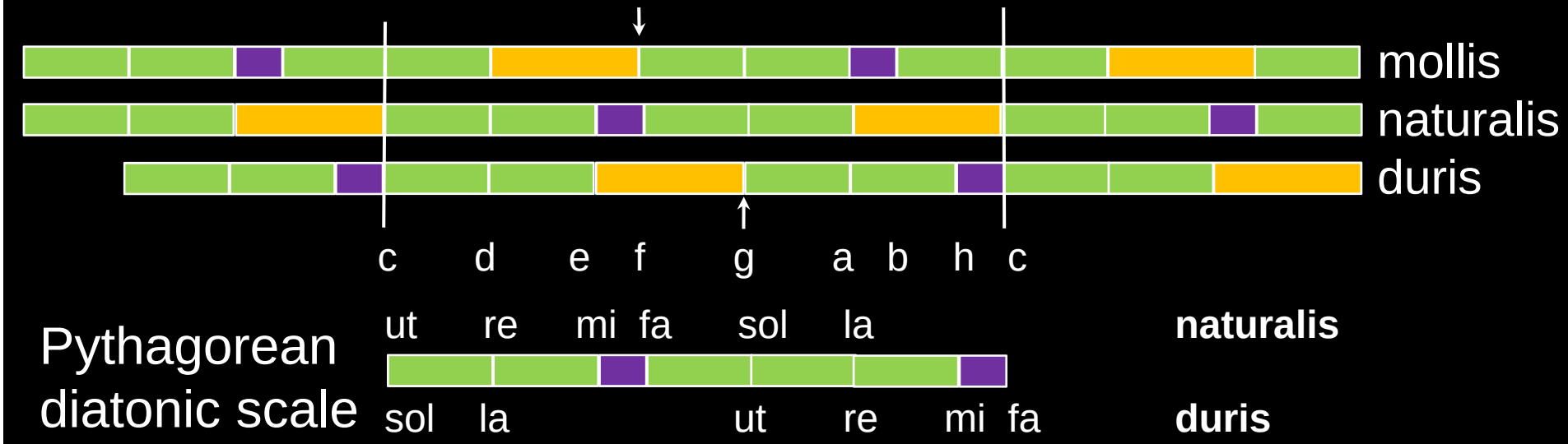
Hexachords





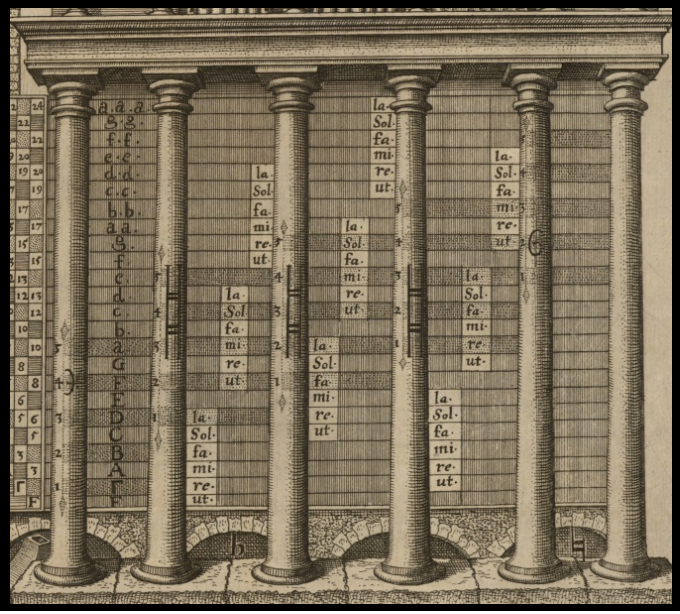
Glarean (1547)
Fludd (1624)

Traditional hexachord system

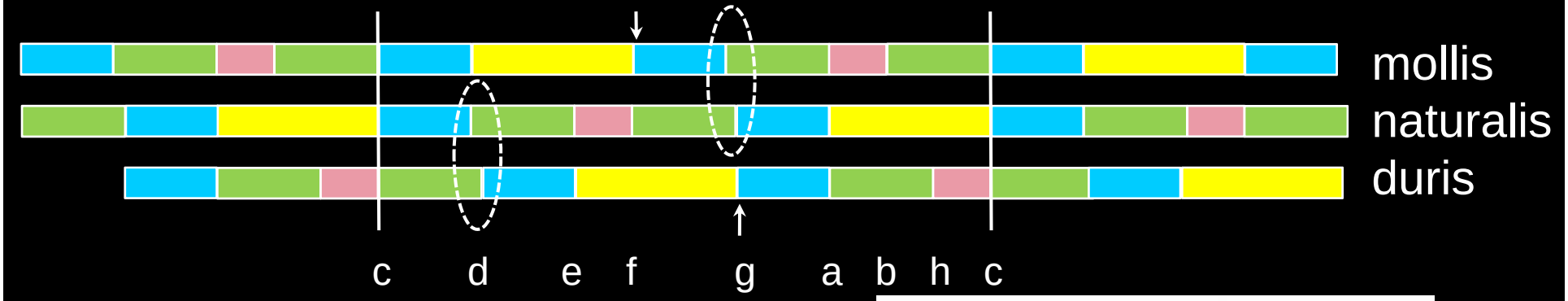


Pythagorean diatonic scale

	tonus	9 : 8
	semitonus	256 : 243
	semiditonus	32 : 27

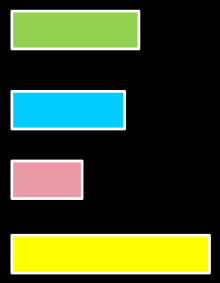


Hexachord system according to Descartes

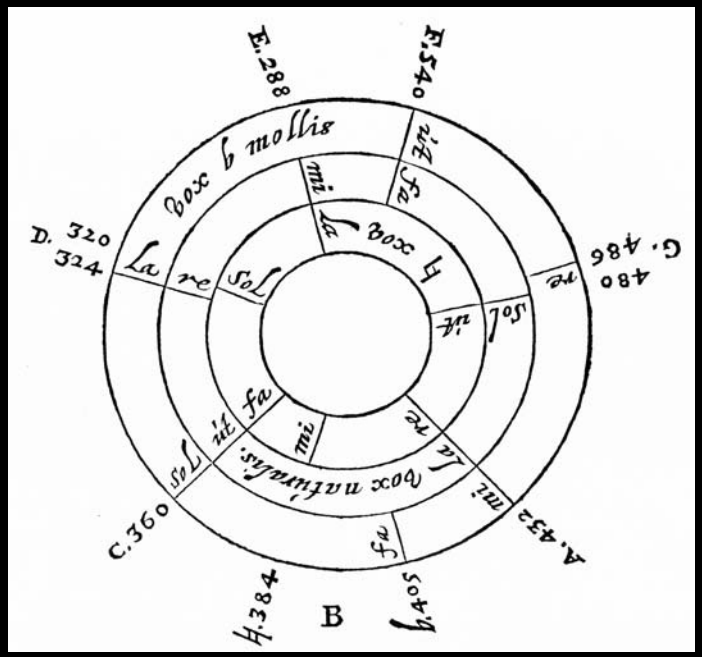


Hexachord:

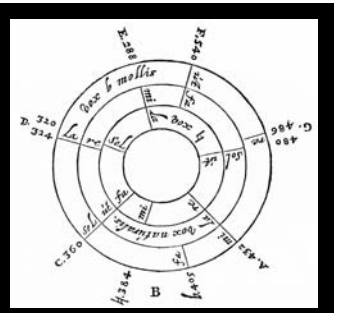
Descartes
Traditional

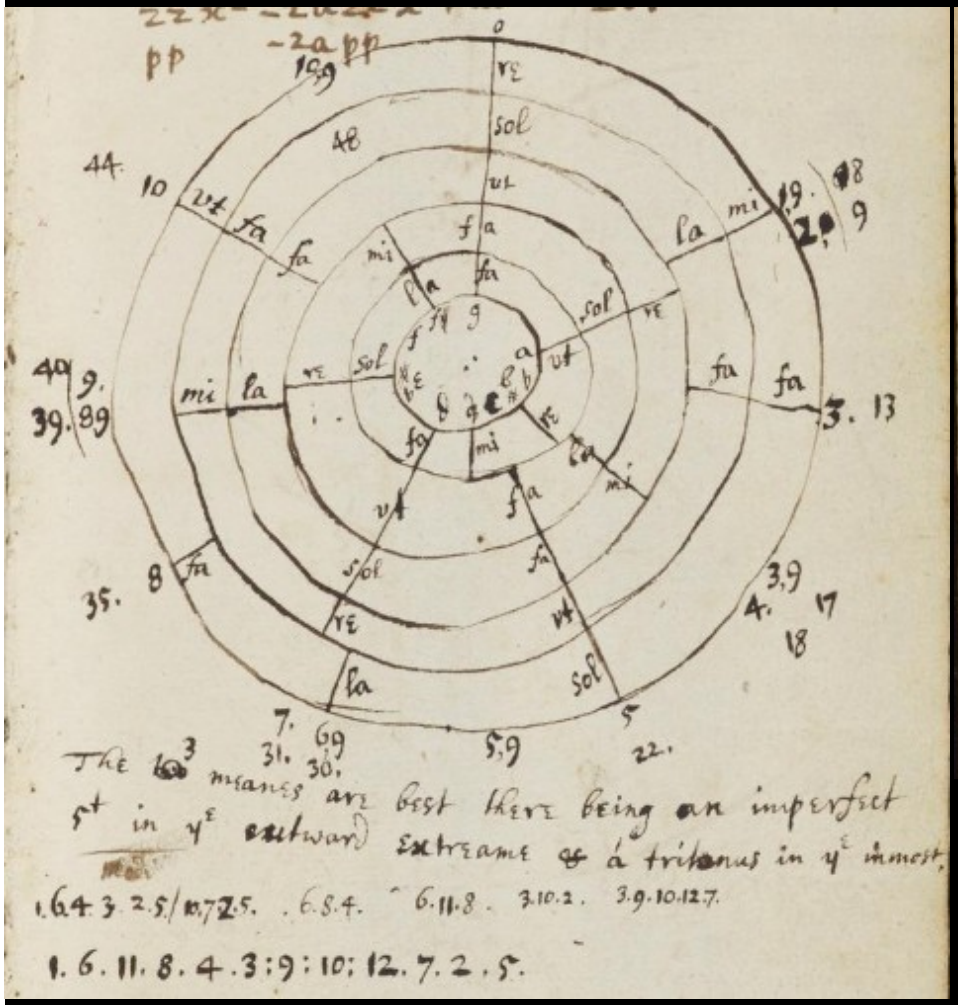


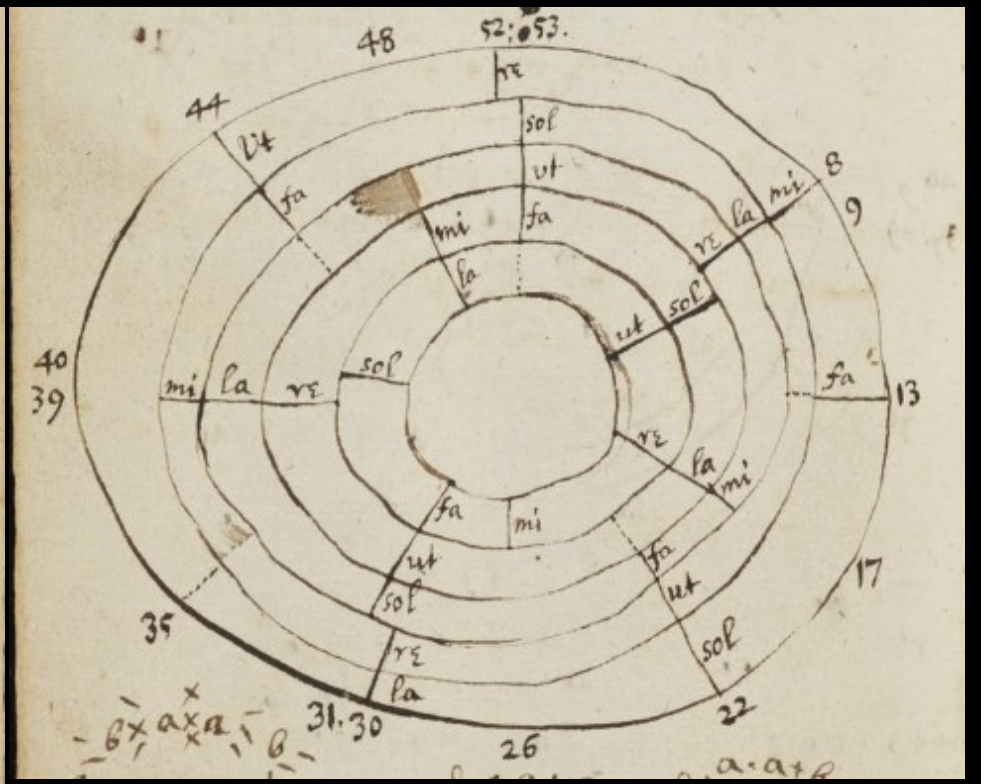
major tone 9 : 8
minor tone 10 : 9
semitone 16 :15
minor third 6 : 5
semitone Pyth. 256 : 243

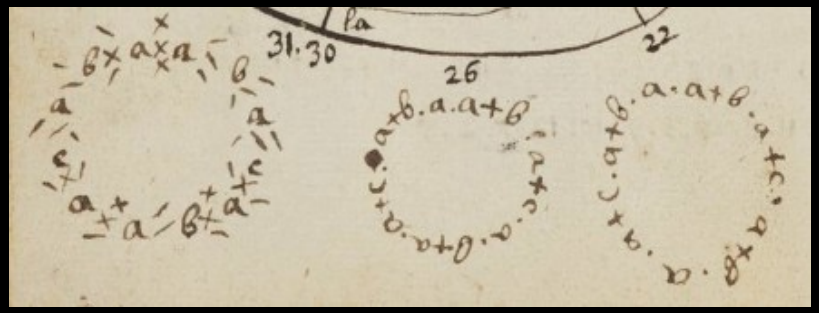


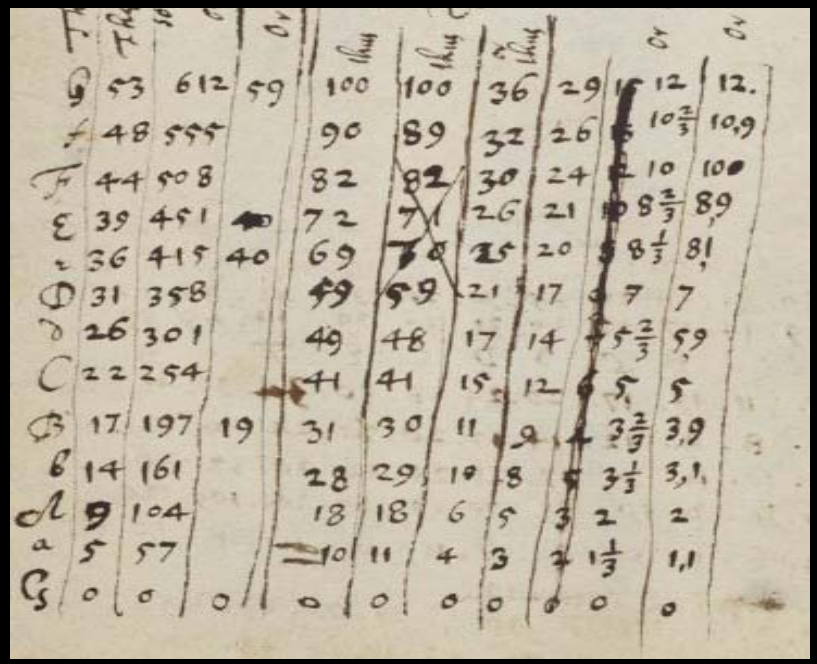
Newton – tuning problem (1665)



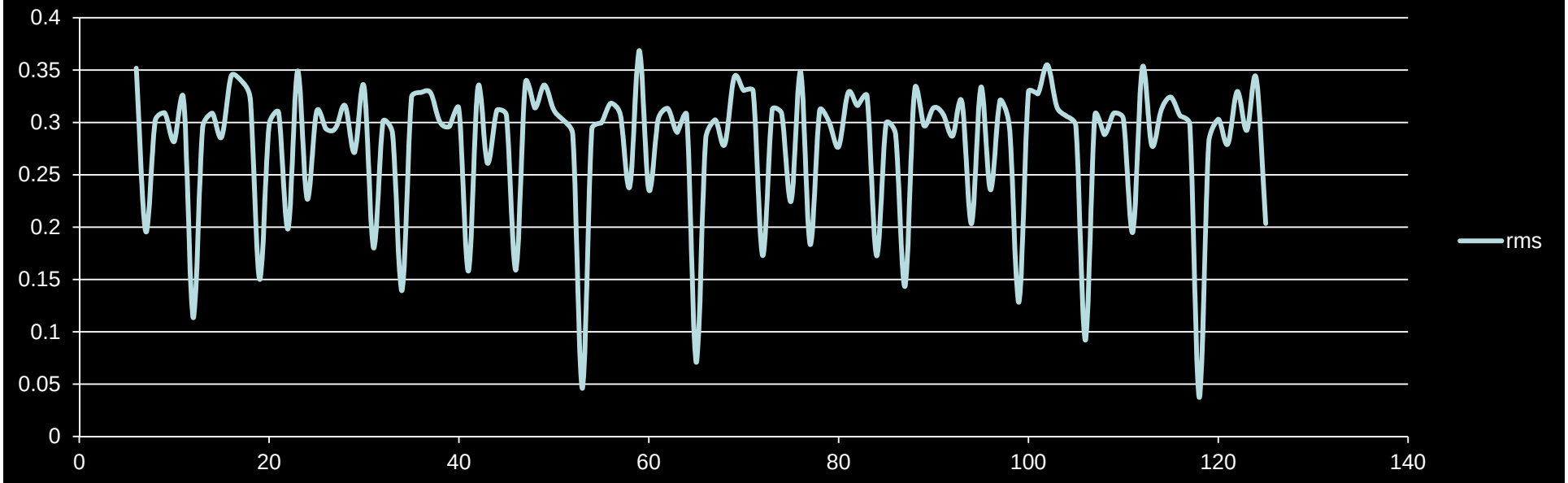








Newton – tuning problem



End