

ZOOMING into CHROMA spaces

Jahreskongress GMTH

Daniel Muzzulini

Michael Dodds

Thomas Noll

Regularity through Incompatibility

PILING TETRACHORDS

Animation

Demo: [Tetrachords](#)

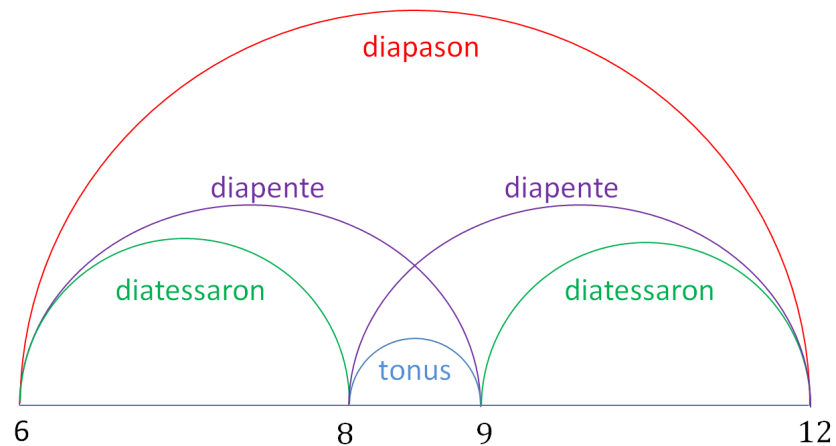
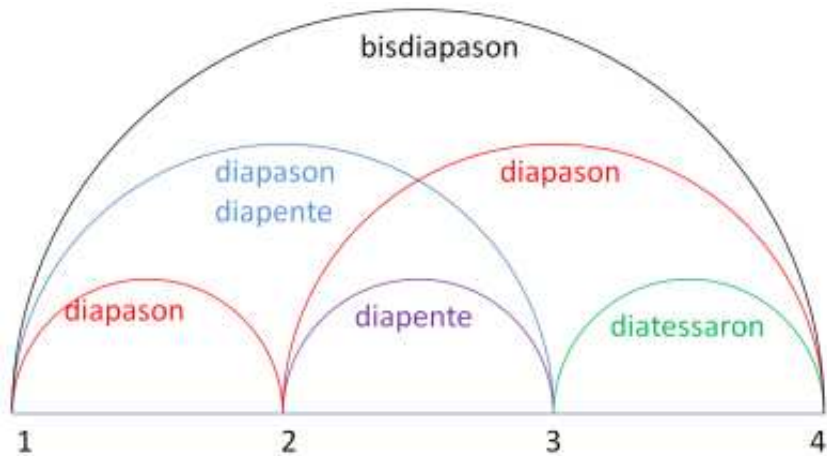
Sequences of octave periodic joined tetrachords
revealing illuminating patterns.

Extended Uses in the 14th Century

TETRAKTYS DIAGRAMS

Tetraktys

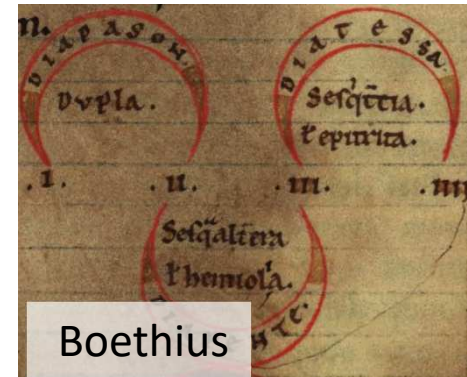
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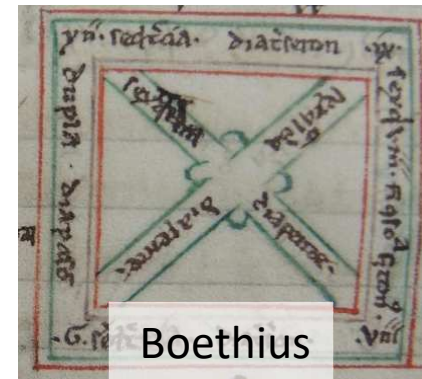
[2]

Jahreskongress GMTH (2021)

daniel@muzzulini.ch



Boethius



Boethius



Guido di Arezzo

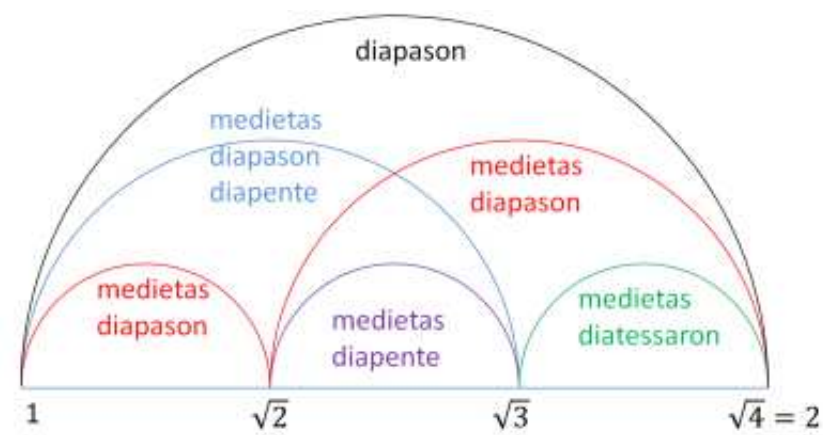
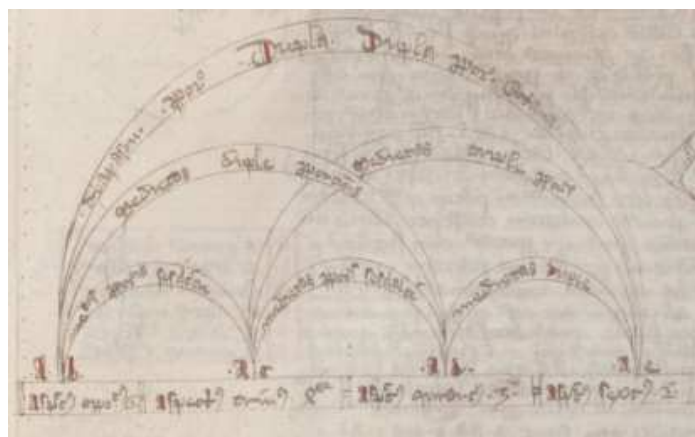


Raffaello

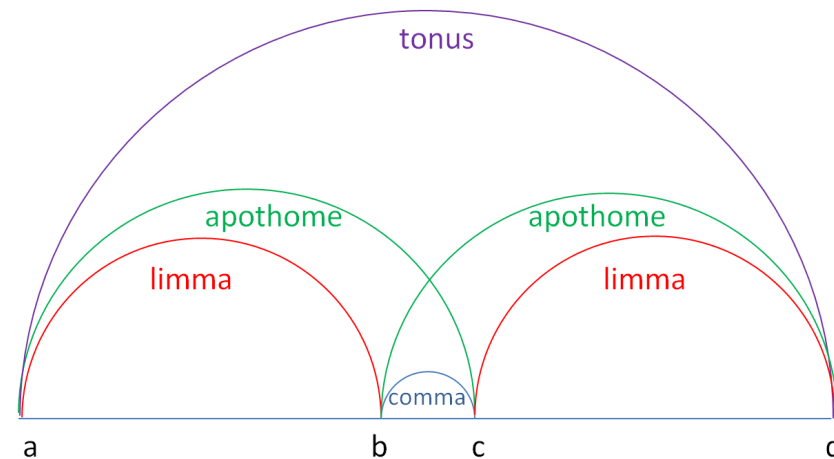
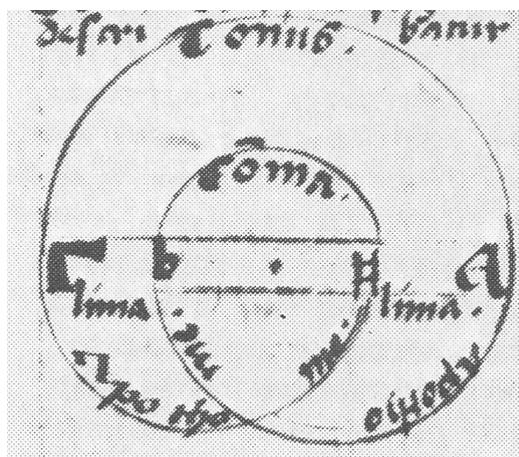
Tetraktys transformations

[3]

Nicole
Oresme



Jacobus
Leodiensis

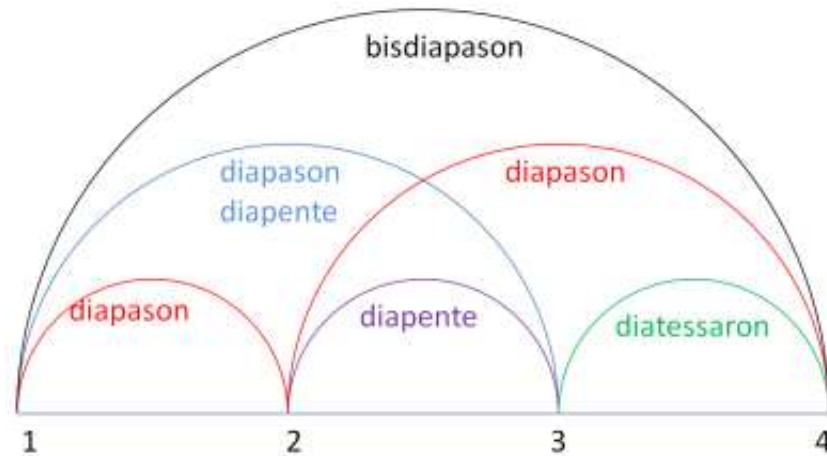


[4]

14th century extensions

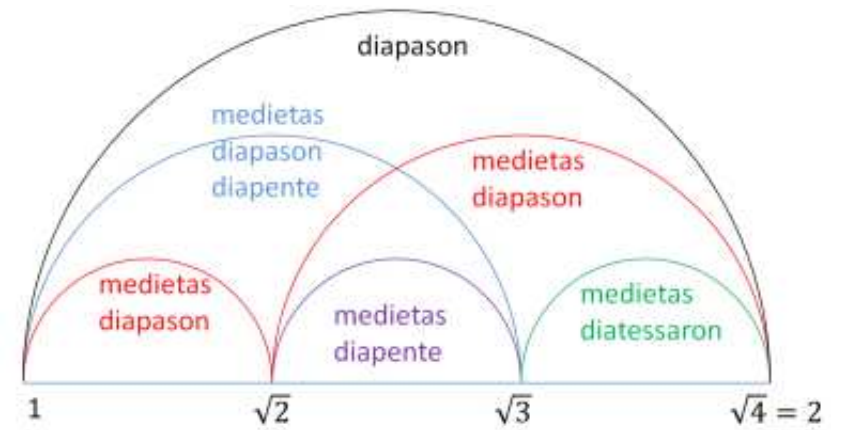
Tetraktys

[1]

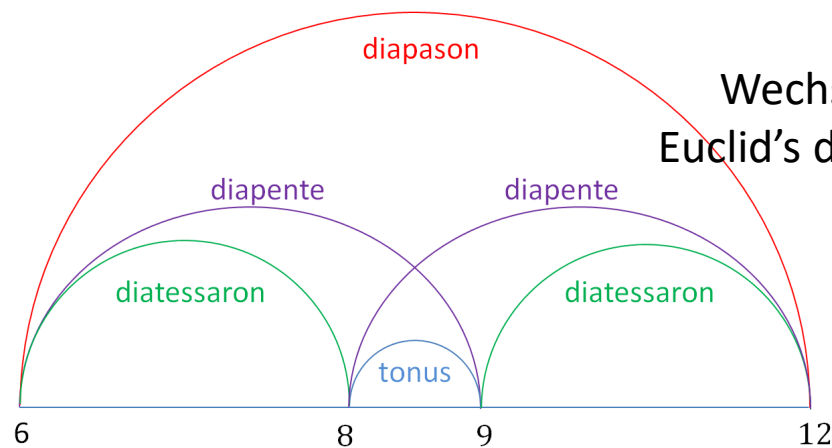


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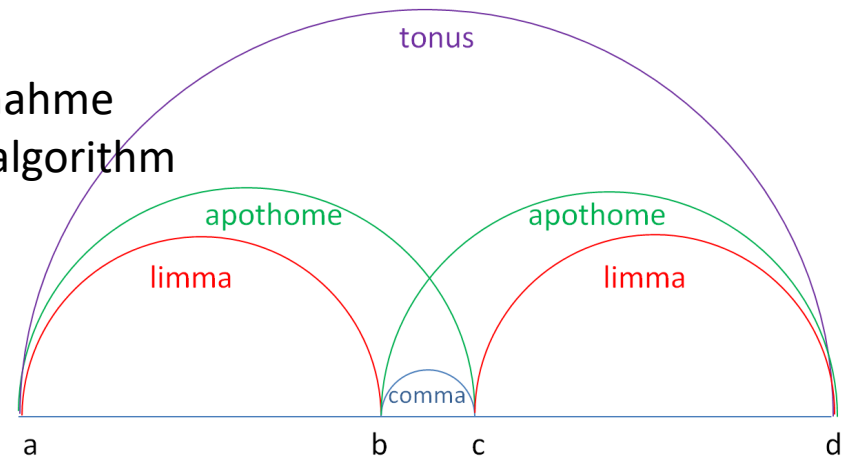
[3]



Wechselwegnahme
Euclid's division algorithm



[2]

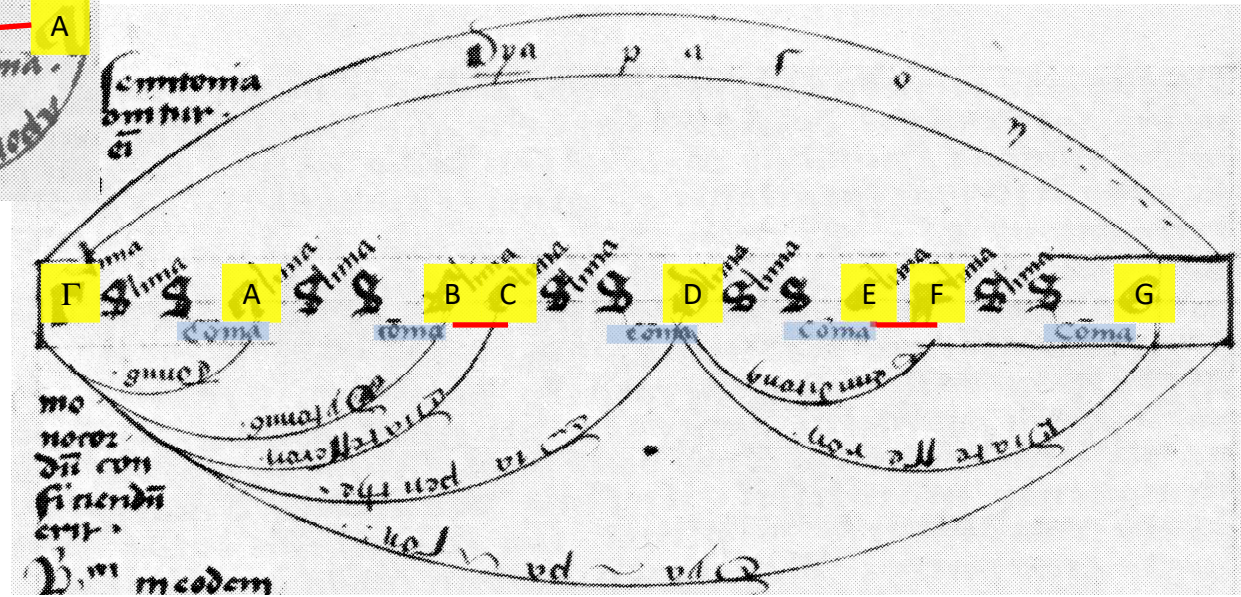
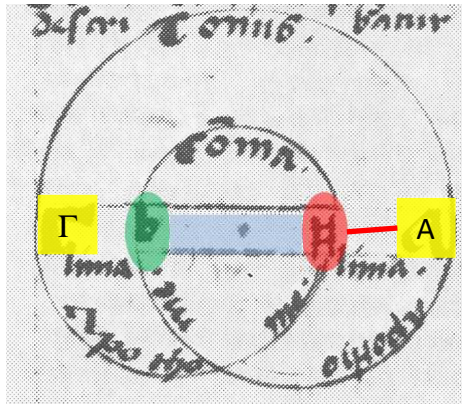


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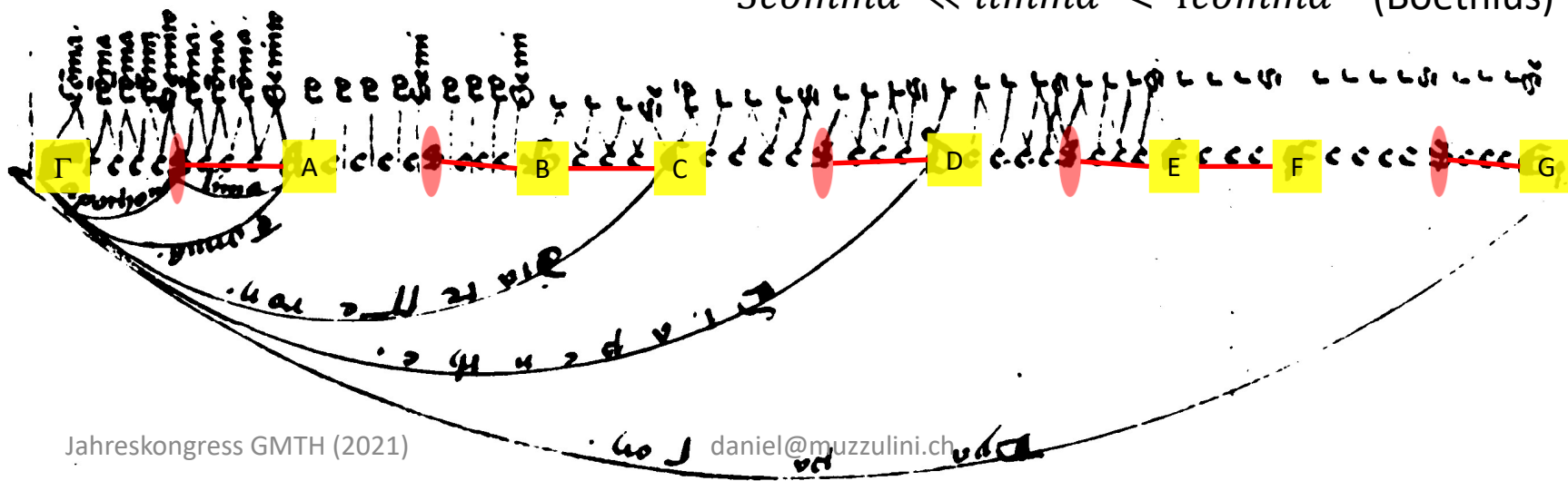
Enharmonic scale – comma scale

JACOBUS LEODIENSIS

Jacobus Leodiensis



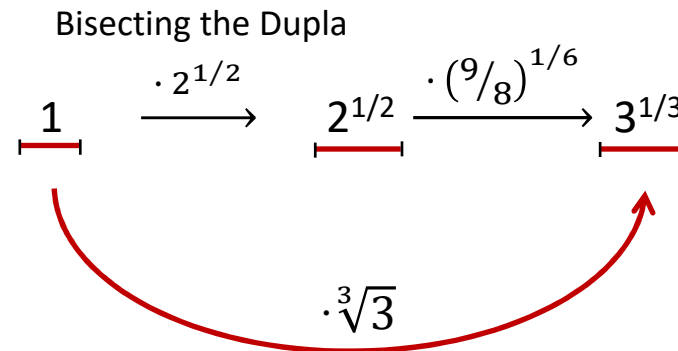
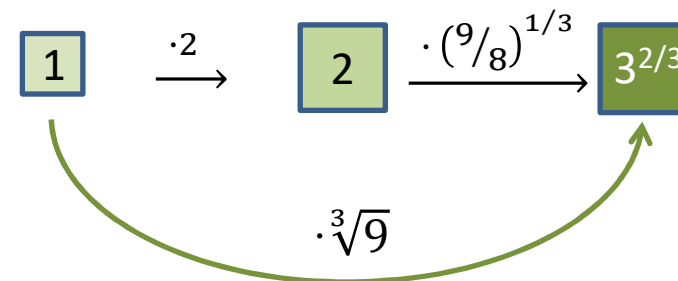
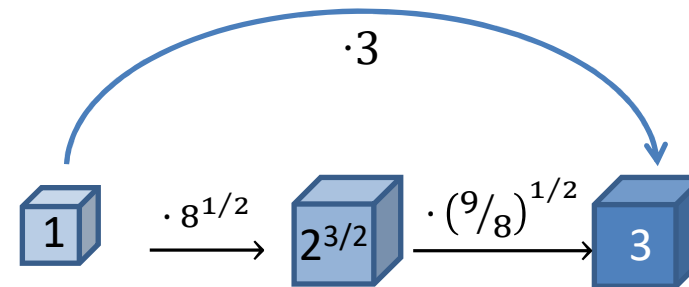
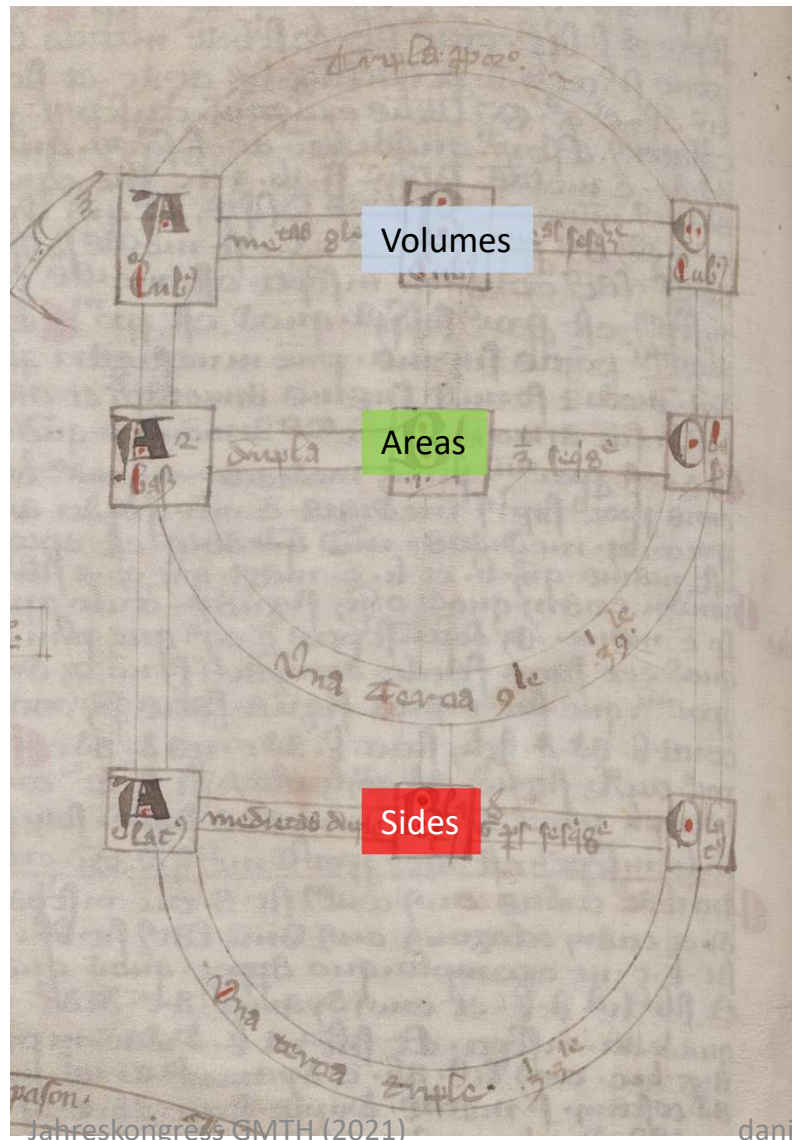
$3\text{comma} \ll \text{limma} < 4\text{comma}$ (Boethius)



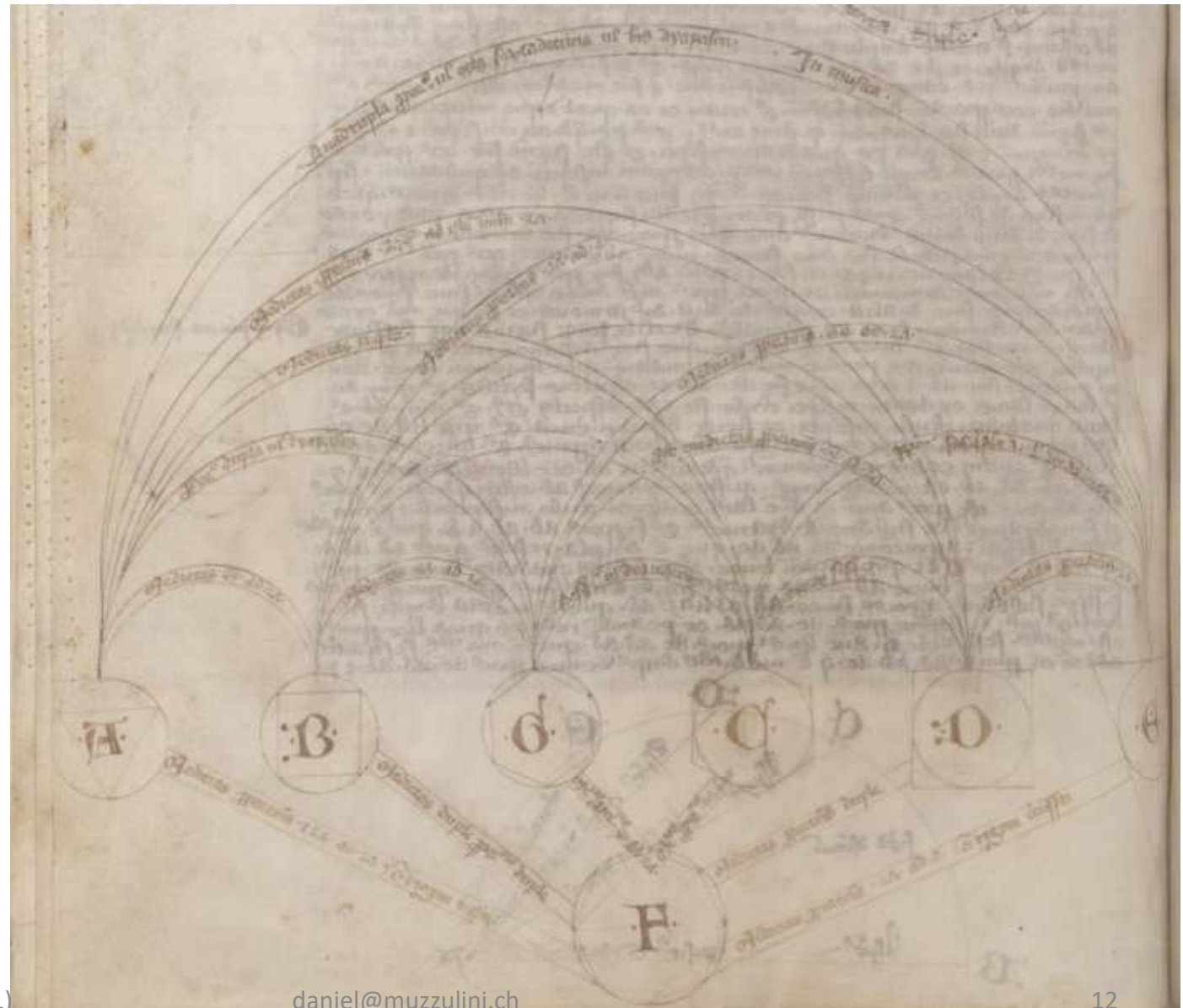
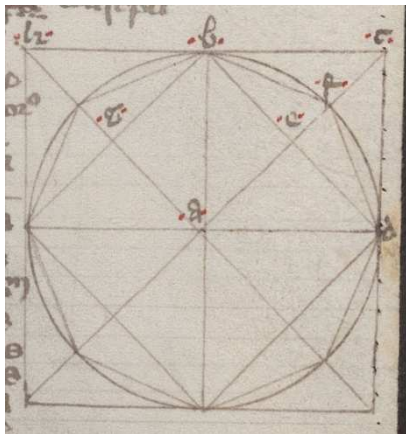
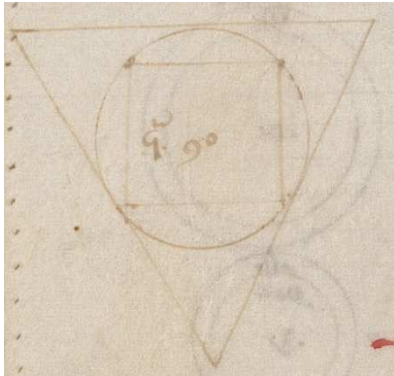
Enriched Pythagorean Harmonics: Fractional Powers

NICOLE ORESME

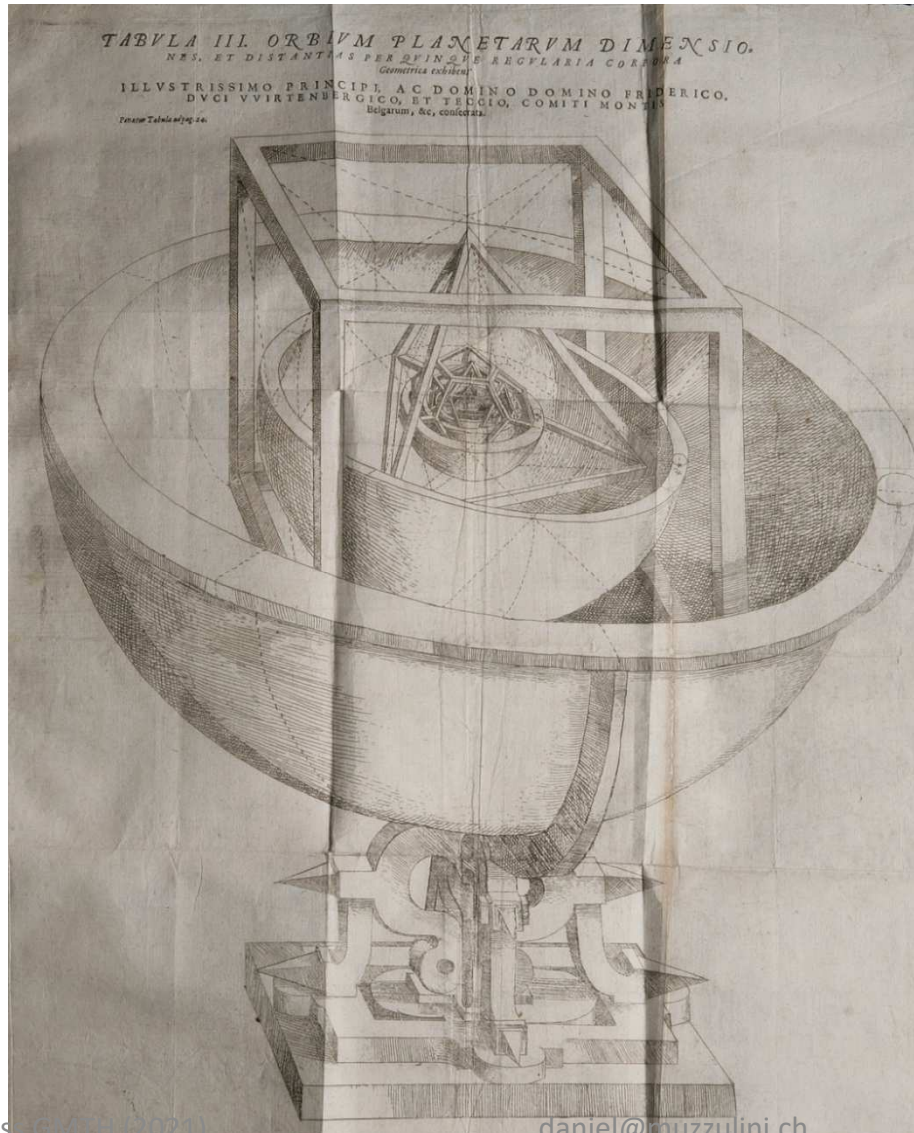
Oresme: Tripling the Cube



Trisecting the Tripla



Johannes Kepler (1596)



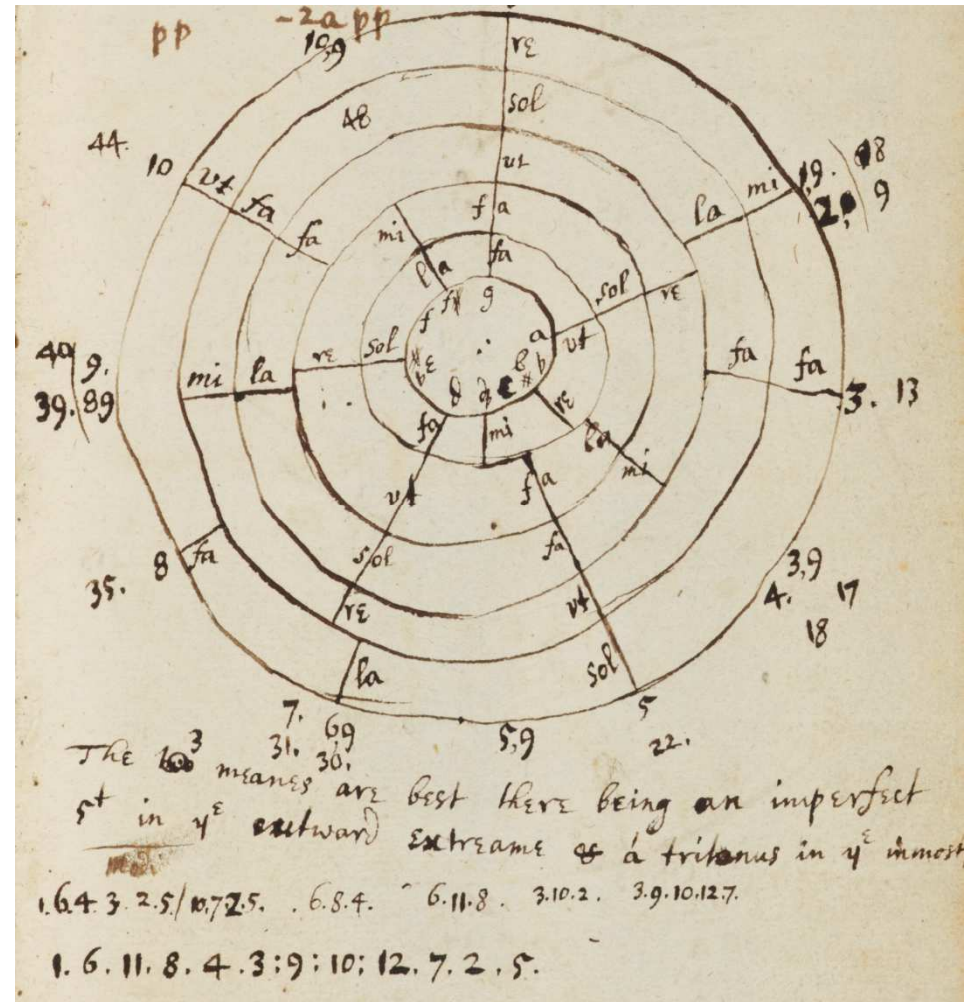
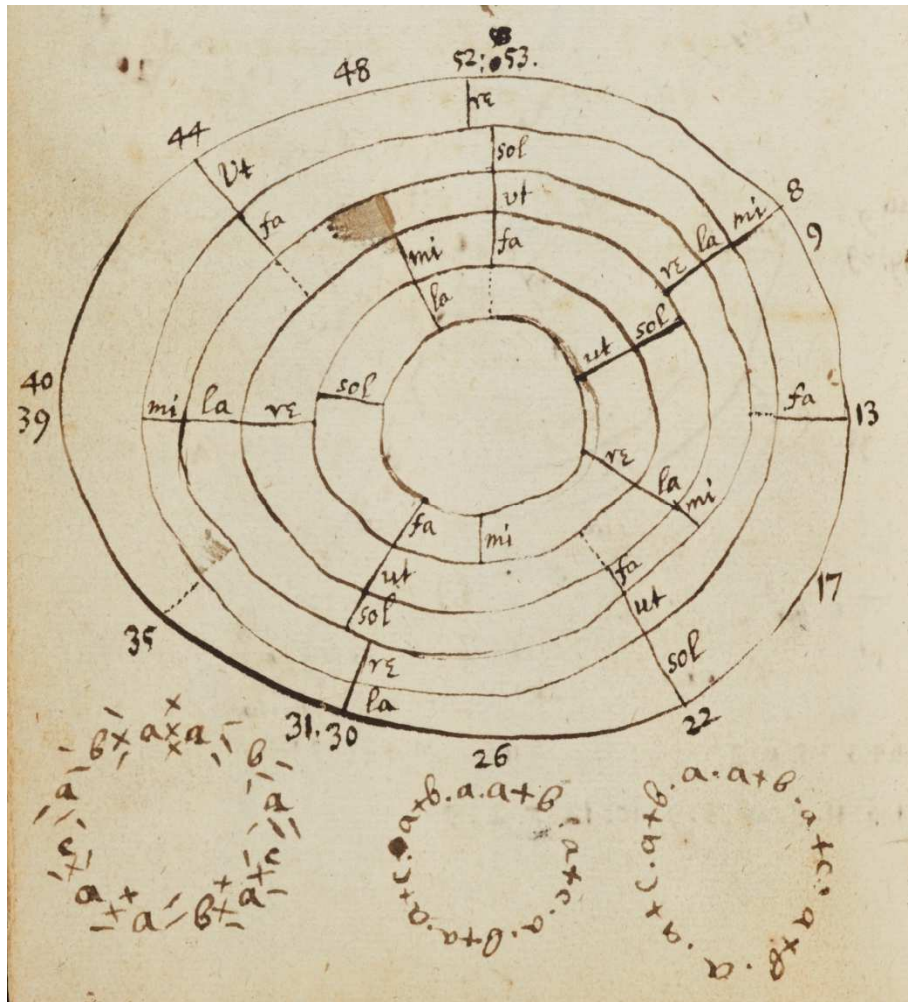
Orbium Planetarum Dimensio.

Harmony of the Spheres
through Platonic Solids
(nested regular polyhedrons)

Syntonic chromatic scale and EDOs

NEWTON

Generalizing Descartes's Hexachords



Logarithms and EDOs

November 20. 1665.

$\frac{1}{2}$	360	2,55630247	2,5563025	360,000000	12,000000 G
$\frac{2}{15}$	384	2,58433118	2,5813883	381,4006678	10,882607 fl.
$\frac{2}{16}$	405	2,60749497	2,6064742	404,086405	9,960909 F
$\frac{3}{9}$	432	2,63548364	2,6319600	428,114581	8,843587 E
$\frac{5}{8}$	450	2,65321247	2,6566458	453,571578	8,136863 E
$\frac{2}{3}$	480	2,68124123	2,6817317	480,542367	7,019550 D
$\frac{32}{49}$	512	2,70926992	2,7068175	509,1168814	5,902237 J
$\frac{3}{4}$	540	2,73239371	2,7319033	539,390559	4,980450 C
$\frac{1}{5}$	576	2,76042244	2,7569892	571,464474	3,863137 B
$\frac{5}{6}$	600	2,77815120	2,7820750	605,445467	3,156413 B
$\frac{8}{9}$	640	2,80617993	2,8071608	641,446973	2,039100 A
$\frac{15}{16}$	675	2,82930381	2,8322467	679,5895149	1,117313 A
1	720	2,85733204	2,8573325	720,000000	0,000000 G

How y^e string 1
720 is to be divided if it may son
all y^e musical notes
of half notes in an
octave
The proportion of
these musical
notes of 12 notes
one to y^e other (viz
of logarithms of y^e
string sounding these
Twelve exact or
quadrant 12 notes
(or y^e logarithms of a
and divided into 12 parts
of each 12 notes being
0,9250858333333333
- 1 just notes being
0,9501716666666666
A string (720) divided
into 12 (geometrically
progression) parts, if
it may sound y^e 12
exact 12 notes in an
octave
The proportion of all
12 musical notes
is right, and exact
if the notes being a
geometric progression.

The notes	The proportion of their sounds	or thus	or thus	thus	How they may be otherwise distinguished by figures	or thus	or thus
G	53	612	59	100	100	36	29
F	48	555		90	89	32	26
E	44	508		82	82	30	24
D	39	451	40	72	71	26	21
C	36	415	40	69	70	25	20
B	31	358	49	59	59	21	17
A	26	301	49	49	48	17	14
G	22	254	41	41	41	15	12
F	17	197	19	31	30	11	9
E	14	161		28	29	10	8
D	9	104		18	18	6	5
C	5	57		10	11	4	3
B	0	0		0	0	0	0

17

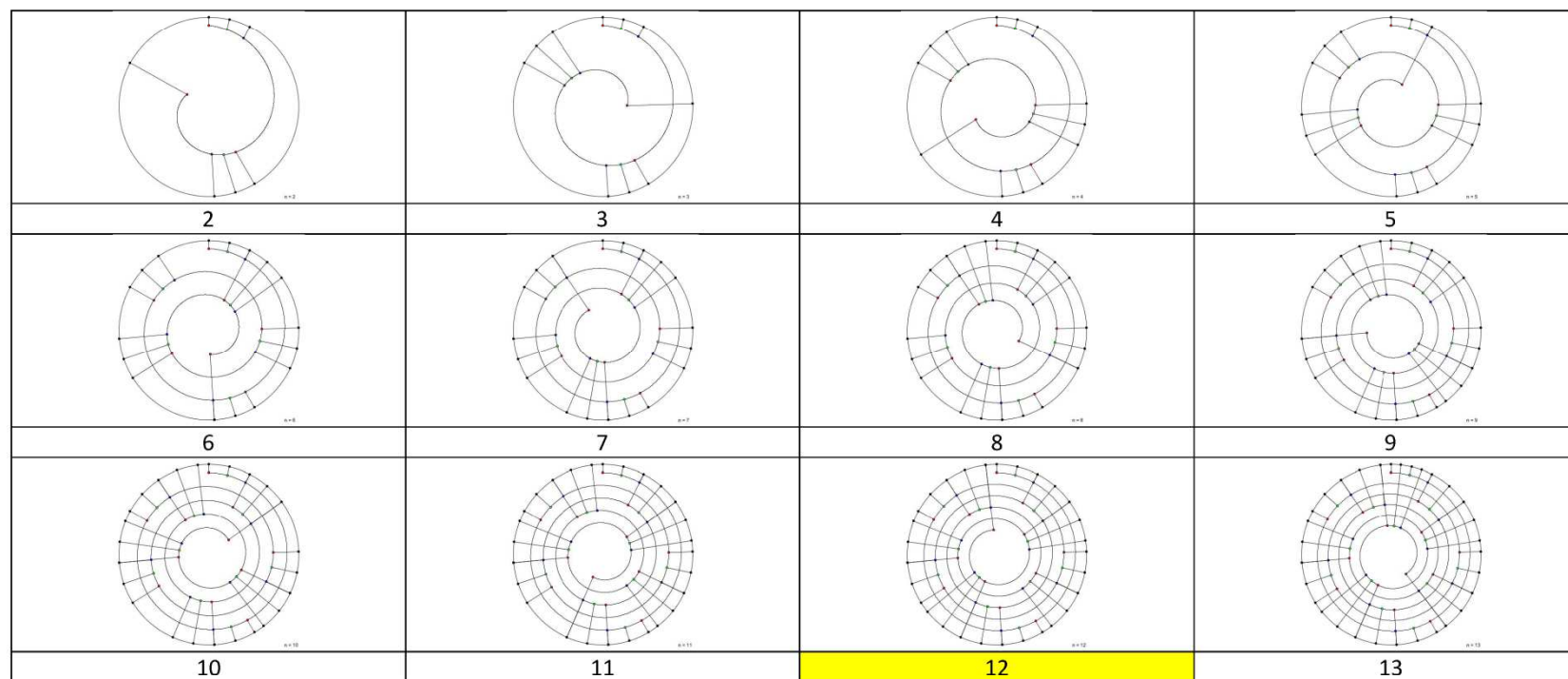
References

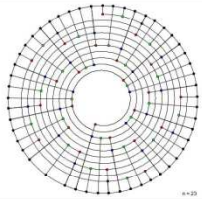
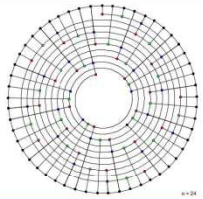
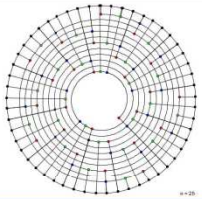
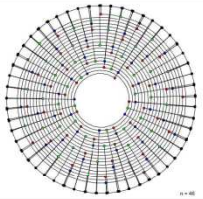
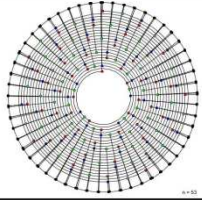
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THANK YOU

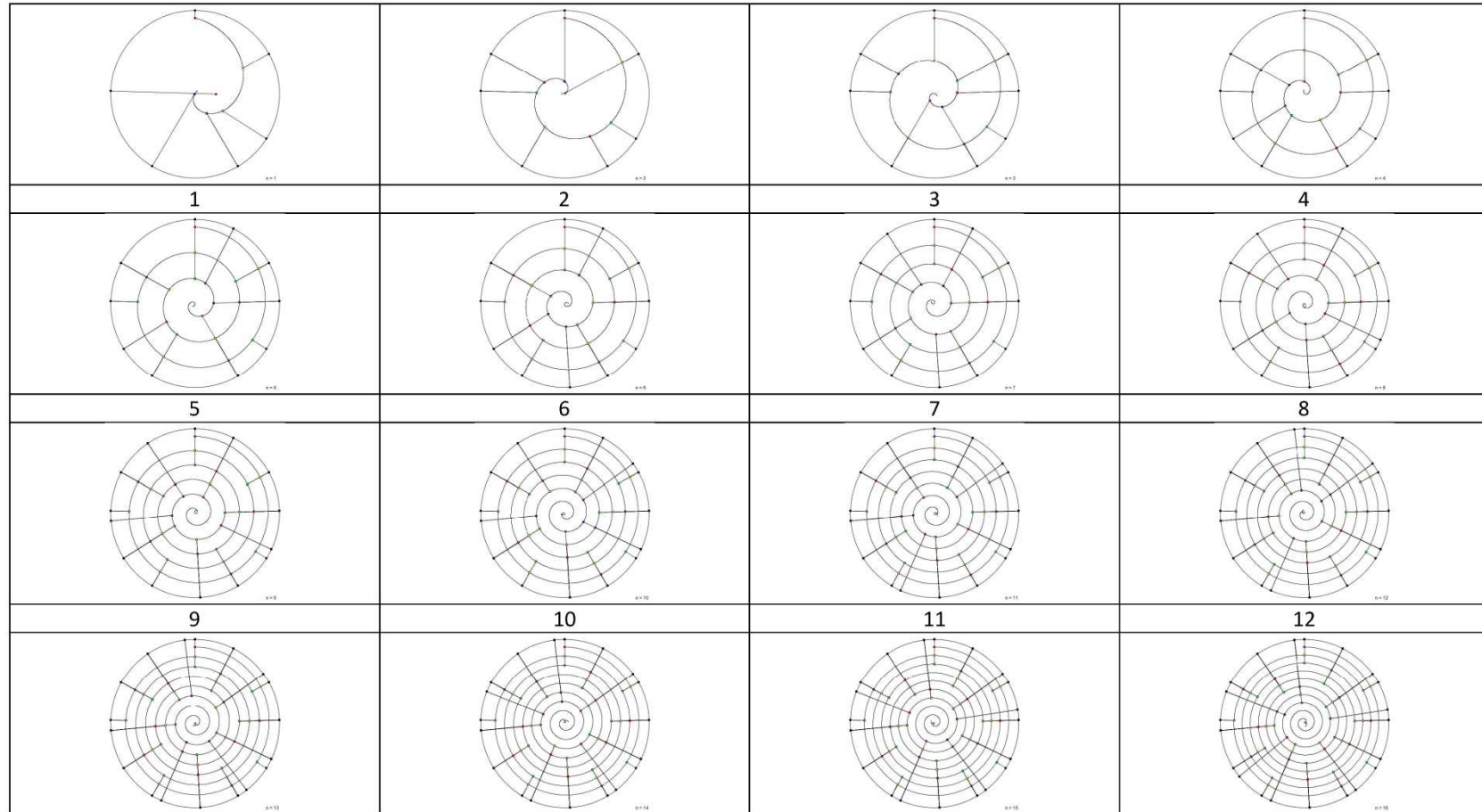
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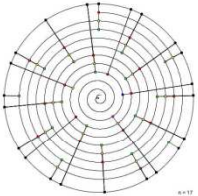
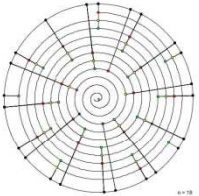
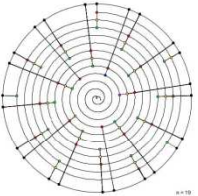
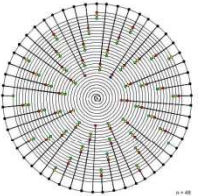
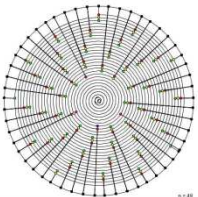
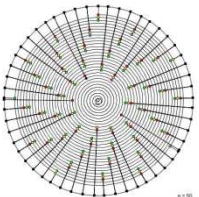
Piling Pythagorean Enharmonic Tetrachords



			
23	24	25	46
			
53			

Piling Pythagorean Hexachords



13	14	15	16
			
17	18	19	48
			
49	50		

Piles of Diatonic Tetrachords (Archytas)

